UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
School of Electrical and Information Engineering
ELEN3013 Signals and Systems IIB

Tutorial 5

Ref 1: Signals, Systems and Transforms by Phillips and Parr, 2nd ed., 1999.

1. Problem 12.6 from Ref 1.

2. Problem 12.12 from Ref 1. Repeat problem 12.12 using an 8 point FFT.

3. Problem 12.13 from Ref 1. Repeat problem 12.13 using a 4, 8 and 16 point FFT.

4. Problem 12.14 from Ref 1. Repeat problem 12.14 using the FFT.

5. Problem 12.18 from Ref 1.

6. Problem 12.19 from Ref 1.

7. Problem 12.20 (a) and (b) from Ref 1.

8. Problem 12.21 from Ref 1.

9. Problem 12.24 from Ref 1.

10. Problem 12.25 from Ref 1.

11 Use the IDFT to obtain the original time sequence from the frequency sequence calculated in problems 2, 3 and 4.

PROBLEMS

12.1. Find the minimum sampling frequency that can be used to obtain samples of each signal listed below. Assume ideal system components.
- **(a)** $v(t) = \sin(200t)$
- **(b)** $w(t) = \sin(100t) - 4\cos(100\pi t) + 30\cos(200t)$
- **(c)** $x(t) = \text{sinc}(200t)$
- **(d)** $y(t) = 50\,\text{sinc}^2(50\pi t)$

12.2. **(a)** Plot the ideally sampled signal and its frequency spectrum for the signal of Problem 12.1(d) for sampling frequencies of 50, 100, and 200 Hz.
(b) Discuss the suitability of these sampling frequencies for the ideal system.

12.3. The signal with the amplitude frequency spectrum shown in Figure P12.3 is to be sampled using an ideal sampler.
(a) Sketch the spectrum of the resulting signal for $|\omega| \leq 120\pi$ rad/s when sampling periods of 10 and 50 ms are used.
(b) Which of the sampling frequencies is acceptable for use if the signal is to be reconstructed using an ideal low-pass filter?

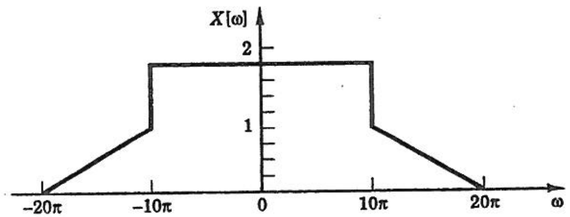

Figure P12.3

12.4. Show that equation (12.13),

$$H(\omega) = \frac{1 - e^{-j\omega T}}{j\omega}$$

is the Fourier transform of the zero-order-hold function with the impulse response,

[eq(12.11)] $$h(t) = u(t) - u(t - T)$$

12.5. **(a)** Use Discrete-Time Fourier Transform Tables 12.1 and 12.2 to find the frequency spectra of the signals listed subsequently.
(b) Plot the magnitude and phase frequency spectra of each of the signal listed below over the frequency range $|\omega| \leq 2\omega_s$.
- **(i)** $f(t) = 8\cos(2\pi t) + 4\sin(4\pi t)$, sampled with $T_s = 0.1$ s.
- **(ii)** $g[n] = 4\cos[0.5\pi n]u[n]$

12.6. Find the discrete-time Fourier transform (DTFT) of each signal shown in Figure P12.6.

12.7. Prove the linearity property of the DFT.

12.8. Prove the frequency-shift property of the DFT.

12.9. Prove the duality property of the DFT.

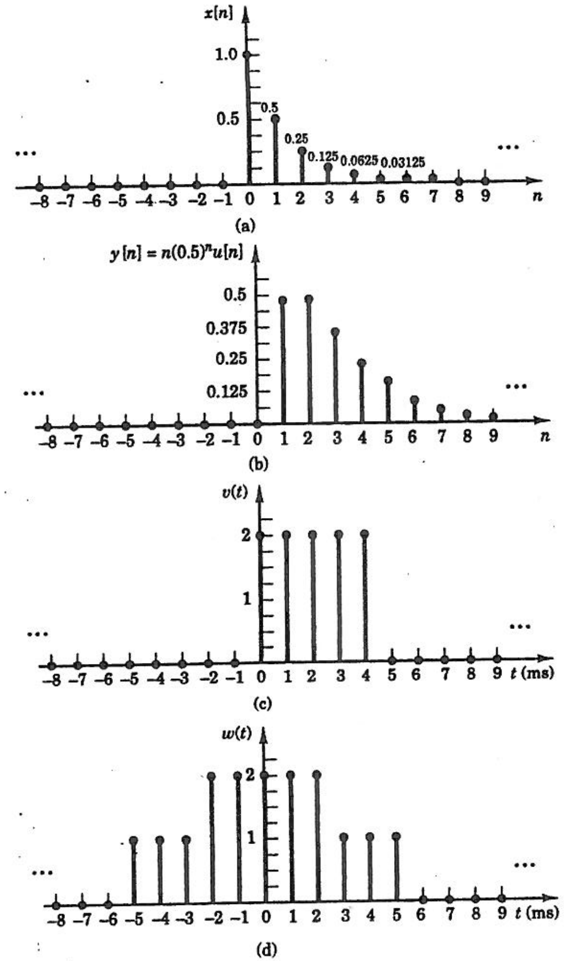

Figure P12.6

12.10. Prove that $\mathcal{DTF}\{nx[n]\} = j\dfrac{dX(\Omega)}{d\Omega}$.

12.11. **(a)** Compute the eight-point DFT of the sequence shown in Fig 12.6(a)
(b) Use MATLAB to confirm the results of part (a).

12.12. **(a)** Compute the eight-point DFT of the sequence shown in Fig 12.6(b)
(b) Use MATLAB to confirm the results of part (a).

12.13. The signal $x(t) = \text{rect}[(t - 2)/4]$ is shown in Figure P12.13.
(a) Compute the four-point DFT of the signal when it is sampled WITH $T_s = 2$ ms PLOT the magnitude and phase spectra.
(b) Use MATLAB to compute the eight-point DFT of the signal WHEN IT IS SAMPLED WITH $T_s = 1$ ms. Plot the magnitude and phase spectra.

(c) Use MATLAB to compute the sixteen-point DFT of the sig when it is sampled with $T_s = 0.5$ milliseconds. Plot the magnitude and phase spectra.
(d) Compare the results of parts (a), (b), and (c). Comment on their relationship.

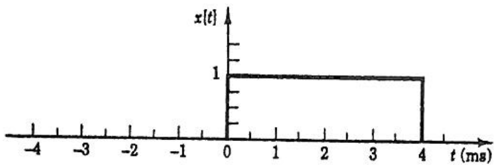

Figure P12.13

12.14. (a) The rudder position sensor of an unlimited hydroplane produces an analog voltage signal, $v(t)$, which is sampled 10 times per second. The first sample is taken at $t = 0$ and the following sequence of values is recorded.

$$v[n] = [0.8, 1.0, 1.2, 1.0, 0.5, 0.1, -0.4, -0.2, 0.5]$$

Use the discrete Fourier transform to determine the range of frequencies that dominate the rudder motion.
(b) Use MATLAB to confirm the results of part (a).

12.15. The signal $x(t) = 5\cos(8\pi t)$ is sampled eight times starting at $t = 0$ with $T = 0.1$ s.
(a) Compute the DFT of this sequence.
(b) Use MATLAB to confirm the results of part (a).
(c) Determine the Fourier transform of $x(t)$ and compare it with the results of part (a) and (b). Explain the differences.

12.16. Repeat Problem 12.15(a) and (b) after multiplying the sequence $x[n]$ by an eight-point Hanning window. Discuss the differences between this DFT and that found in Problem 12.15.

12.17. (a) The table below gives the four-point DFT of a discrete-time sequence. Find $x[n]$.

k	$\lvert X(k)\rvert$	$\angle X(k)$
0	20.000	0
1	8.4853	$-3\pi/4$
2	0.0000	0
3	8.4853	$3\pi/4$

(b) Use MATLAB to confirm the results of part (a).

12.18. An analog signal of 1 s duration is sampled at 512 equally spaced times and its DFT is computed.
(a) What is the separation in rad/s between successive frequency components?
(b) What is the highest frequency present in the spectrum?
(c) What is the highest frequency that can be allowed in the analog signal if aliasing is to be prevented?

12.19. The of the analog signal $f(t) = 7\cos(100t)\cos(40t)$ is to BE COMPUTED
(a) What is the minimum sampling frequency to avoid aliasing?
(b) If a sampling frequency of $\omega_s = 300$ rad/s is used, how MANY SAMPLES MUST BE taken to give a frequency resolution of 1 rad/s?

12.20. Given the two four-point sequences

$$x[n] = [-2, -1, 0, 2] \quad \text{and} \quad y[n] = [-1, -2$$

find:
(a) $x[n]*y[n]$, the linear convolution
(b) $x[n]\circledast y[n]$, the circular convolution
(c) $R_{xy}[p]$, the cross-correlation of $x[n]$ and $y[n]$
(d) $R_{yx}[p]$, the cross-correlation of $y[n]$ and $x[n]$
(e) $R_{xx}[p]$, the autocorrelation of $x[n]$
(f) Use MATLAB to confirm the results of parts (a) through (

12.21. Extend the sequences given in Problem 12.20 by zero paddi convolution on the extended sequences so that the result equ ear convolution of the original sequences.

12.22. The four-point DFTs of $x[n]$ and $h[n]$ are

$$X[k] = [8, -2 - j2, 0, -2 + j2]$$

and

$$H[k] = [1.89, 0.75 - j0.64, 0.61, 0.75 + $$

Find the value of $y[2]$, where $y[n] = x[n]\circledast h[n]$.

12.23. The four-point DFTs of two discrete-time signals are given be

$$X[k] = [22, -4 + j2, -6, -4 - j2]$$

$$Y[k] = [8, -2 - j2, 0, -2 + j2]$$

(a) If $v[n] = x[n]*y[n]$, find $v[2]$.
(b) If $w[n] = x[n]\circledast y[n]$, find $w[2]$.
(c) Find $R_{xy}[2]$.
(d) Find $R_{yx}[2]$.
(e) Find $R_{xx}[2]$.

(f) Calculate the periodogram spectral estimate of the signal

12.24. (a) Draw the four-point FFT signal flow diagram and use it to SOLVE FOR THE DFT OF THE sequence shown in Figure P12.24.
(b) Use MATLAB to confirm the results of part (a).

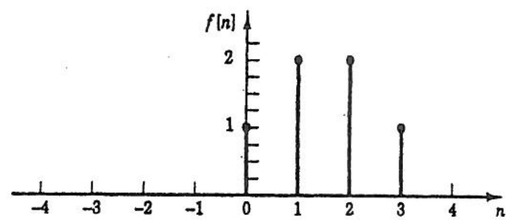

Figure P12.24