UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

School of Electrical and Information Engineering

ELEN3013 Signals and Systems IIB

Tutorial 3

Ref 1: Signals, Systems and Transforms by Phillips and Parr, 2nd ed., 1999.

1. Problem 11.1 from Ref 1. Determine the z transforms from first principles in addition to tables.

2. Problem 11.2 from Ref 1. Determine the z transform from first principles in addition to tables. Determine the z transform from the Laplace function for each time signal.

3. Problem 11.6 from Ref 1. Determine the z transform.

4. Problem 11.7 from Ref 1.

5. Problem 11.9 from Ref 1.

6. Problem 11.10 from Ref 1.

7. Problem 11.11 from Ref 1.

8. Problem 11.12, part (a) from Ref 1.

9. Problem 11.14 part (a) from Ref 1.

10. Problem 11.17 from Ref 1.

11. Problem 11.19 from Ref 1.

12. Problem 11.23 from Ref 1.

13. A system is given by

$$y(k+2) + 0.5y(k+1) + 0.8y(k) = x(k+2) + 2x(k)$$

Using the z transform,

(a) Draw a block diagram of the system.

(b) Determine the system transfer function.

(c) Determine the impulse response of the system.

(d) Determine the unit step response of the system.

(e) Determine the zero state, zero input and total output response of the system for an input

$$x(k) = k^2 u(k)$$

and initial conditions $y(0) = 1$, $y(1) = 3$. $u(k)$ is the unit step function.

(f) Is the system stable? Give a mathematical argument to support your answer.

(g) Draw a Bode diagram of the system frequency response.

SUMMARY

The unilateral and bilateral z-transforms were introduced in this chapter. The unilateral transform is used in the analysis and design of linear time-invariant (LTI) discrete-time system that are causal. This transform is especially useful in understanding the characteristics and in the design of causal digital filters. The unilateral transform is also useful in the analysis of switched LTI discrete-time systems, since this transform allows us to include initial conditions.

The bilateral z-transform is useful in the steady-state analysis of LTI discrete-time systems, and in the analysis and design of noncausal systems. Recall that noncausal systems are not realizable in real time. However, noncausal systems are realizable for the digital processing of recorded signals.

The next chapter involves the discrete-time Fourier transform. This transform is the result of applying the Fourier techniques of Chapters 4 through 6 to discrete-time signals, especially to sampled signals.

REFERENCES

1. G. Doetsch, *Guide to the Applications of the Laplace and z-Transforms*. New York: Van Nostrand Reinhold, 1971.
2. E. I. Jury, *Theory and Application of the z-Transform Method*. New York: Krieger, 1973.
3. C. L. Phillips and H. T. Nagle, *Digital Control System Analysis and Design*, 3rd ed. Upper Saddle River, NJ: Prentice Hall, 1996.

PROBLEMS

11.1. Express the unilateral z-transforms of the following functions as rational functions. Tables may be used.
(a) 0.8^n **(b)** $0.9^n + 3(1.1)^n$
(c) $3e^{-0.1n}$ **(d)** $3e^{-j0.1n}$
(e) $\cos 2n$ **(f)** $20\cos(2n - \pi/4)$
(g) $10e^{-0.2n}\cos 0.5n$ **(h)** $10e^{-0.2n}\cos(0.5n - \pi/4)$

11.2. The signals given are sampled every 0.05 s, beginning at $t = 0$. Find the unilateral z-transforms of the sampled functions, with each transform expressed as a rational function.
(a) e^{-t} **(b)** $2e^{-t} + 3e^{t}$
(c) $3e^{-0.1t}$ **(d)** $3e^{-jt}$
(e) $\cos(t)$ **(f)** $10e^{-t}\cos(t)$

11.3. The signal $x(t) = e^{-2t}$ is to be sampled at a rate of five samples per time constant.
(a) Find the required sample period.
(b) Find $x(nT)$.
(c) Find the z-transform of $x(nT)$.

11.4. **(a)** The signal e^{-5t} is sampled every 0.2 s, beginning at $t = 0$. Find the z-transform of the sampled signal.
(b) The signal e^{-t} is sampled every second, beginning at $t = 0$. Find the z-transform of the sampled signal.

(c) Why are the z-transforms found in parts (a) and (b) identical?
(d) A third function e^{at} is sampled every T seconds. Find two different such that the z-transforms of the sampled function are identical t(
(a) and (b).

11.5. The signal $x(t) = \sin \pi t$ is sampled with the sample period $T = 1$ s.
(a) Find the number of samples per period of the sinusoid.
(b) Find $X(z)$.
(c) Why does $X(z)$ have the unusual value found in part (b), because
(d) Find a different nonzero signal that has the same z-transform as fo
(e) Find a different sample period that gives the same $X(z)$ as found i

11.6. **(a)** Use the z-transform to evaluate the following series.
(i) $x = \sum_{n=0}^{\infty} 0.5^n$ **(ii)** $x = \sum_{n=2}^{\infty} 0.5^n$
(b) Use the z-transform to evaluate the series

$$x = \sum_{n=0}^{\infty} 0.5^n \cos(0.1n)$$

11.7. **(a)** A function $f[n] = A\cos(\Omega n)$ has the z-transform

$$F(z) = \frac{3z(z - 0.6967)}{z^2 - 1.3934z + 1}$$

Find A and Ω.
(b) A function $f(t) = A\cos(\omega t)$ is sampled every $T = 0.0001$ s, beginning at $t=0$. The z-transform of the sampled function is given in part (a). Find A and ω

11.8. The z-transform of a discrete-time function $f[n]$ is given by

$$F(z) = \frac{z^2}{(z-1)(z+1)}$$

(a) Apply the final-value property to $F(z)$.
(b) Check your result in part (a) by finding the inverse z-transform of $F(z)$
(c) Why are the results of parts (a) and (b) different?

11.9. **(a)** Given $\mathcal{Z}[2^n] = z/(z-2)$. Find the z-transform of $f[n]$ as given, using only the properties of z-transforms.

$$f[n] = n(n-1)2^n u[n]$$

(b) Verify the results of part (a) using the z-transform table.

11.10. A function $y[n]$ has the unilateral z-transform

$$Y(z) = \frac{z^3}{z^3 - 3z^2 + 5z - 9}$$

(a) Find the z-transform of $y_1[n] = y[n-3]u[n-3]$.
(b) Find the z-transform of $y_2[n] = y[n+3]u[n]$.
(c) Evaluate $y[n]$ for $n = 0$ and 3, $y[n-3]u[n-3]$ for $n = 3$, and $y[n+3]u[n]$ for $n = 0$ by expanding the appropriate z-transform into power serie:
(d) Are the values found in part (c) consistent? Explain why.

11.11. Given: $f[n] = a^n u[n]$. Find the z-transforms of
(a) $f[n/2]$

(b) $f[n-2]u[n-2]$; verify your result by a power-series expansion
(c) $f[n+2]u[n]$; verify your result by a power-series expansion
(d) $b^n f[n]$, using two different procedures

11.12. **(a)** Given the unilateral z-transforms below. Find the inverse z-transform of each function.

(i) $X(z) = \dfrac{0.4z^2}{(z-1)(z-0.6)}$

(ii) $X(z) = \dfrac{0.4z}{(z-1)(z-0.6)}$

(iii) $X(z) = \dfrac{0.4}{(z-1)(z-0.6)}$

(iv) $X(z) = \dfrac{z}{z^2 - z + 1}$

(b) Verify the partial-fraction expansions in part (a) using MATLAB.
(c) Evaluate each $x[n]$ in part (a) for the first three nonzero values.
(d) Verify the results in part (c) by expanding each $X(z)$ in part (a) into a power series using long division.
(e) Use the final-value property to evaluate $x[\infty]$ for each function in part (a).
(f) Check the results of part (e), using each $x[n]$ found in part (a).
(g) Use the initial-value property to evaluate $x[0]$ for each function in part (a).
(h) Check the results of part (g), using each $x[n]$ found in part (a).

11.13. Consider the transforms from Problem 11.12.

$$X_1(z) = \frac{0.4z^2}{(z-1)(z-0.6)}$$

$$X_2(z) = \frac{0.4z}{(z-1)(z-0.6)}$$

$$X_3(z) = \frac{0.4}{(z-1)(z-0.6)}$$

(a) Without calculating the inverse transforms, state how $x_1[n]$, $x_2[n]$, and $x_3[n]$ are related.
(b) Verify the results of part (a) by finding the inverse transforms.
(c) Verify the partial-fraction expansions in part (b) using MATLAB.

11.14. Given the following difference equation and excitation:

$$y[n] - 1.5y[n-1] + 0.5y[n-2] = x[n]$$

$$x[n] = \begin{cases} 1, & n = 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find $y[n]$, using the z-transform.
(b) Check the partial-fraction expansions in part (a) using MATLAB.
(c) Verify the results of part (a) for $n = 0, 1, 2, 3$, and 4, by solving the difference equation by iteration.
(d) Use MATLAB to check the results in part (c).
(e) Verify the value of $y[0]$ in part (a), using the initial-value property.
(f) Will the final-value property give the correct value of $y[\infty]$? If your answer is yes, find the final value. Otherwise, state why the final-value property is not applicable.

11.15. Given the following difference equation and excitation:

$$y[n] - 0.75y[n-1] + 0.125y[n-2] = x[n]$$

$$x[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find $y[n]$, using the z-transform.
(b) Check the partial-fraction expansions in part (a) using MATLAB.
(c) Verify the results of part (a) for $n = 0, 1, 2, 3$, and 4, by solvir equation by iteration.
(d) Use MATLAB to check the results in part (c).
(e) Verify the value of $y[0]$ in part (a), using the initial-value proper
(f) Will the final-value property give the correct value of $y[\infty]$? If yo find the final value. Otherwise, state why the final-value property

11.16. **(a)** The difference equation given models an LTI system. Find the system TRANSFER FUNCTION

$$y[n] - 0.75y[n-1] + 0.125y[n-2] = x[n]$$

(b) Find the unit step response for the system of part (a).
(c) Use MATLAB to check the partial-fraction expansion in part (b).
(d) From part (b), give the values of $y[0]$, $y[1]$, and $y[2]$.
(e) Verify the results of part (d) by solving the difference equation il
(f) Verify the results in part (e) using MATLAB.

11.17. **(a)** The difference equation given models an LTI system. Find the system TRANSFER FUNCTION

$$y[n] - y[n-1] + 0.5y[n-2] = x[n]$$

(b) Find the unit step response for this system.
(c) Evaluate $y[n]$, $n = 1, 2, 3, 4$, using the results of (b).
(d) Verify the results of (c) using the system difference equation.

11.18. **(a)** Given the system described by the difference equation

$$y[n] - 0.95y[n-1] = 0.1x[n]$$

Find the steady-state value of the output for the unit step input x
(b) Verify the results in (a) by finding $y[n]$ for all n.
(c) Verify the results in (a) using MATLAB to solve the system differe
(d) Repeat (a) for the input $x[n] = \cos(\pi n/2)$
(e) Verify the results in (d) using (10.83).

11.19. Consider the block diagram of a discrete-time system given in Figure P11.19
(a) Find the difference-equation model of this system.
(b) Find the system transfer function.
(c) Determine the range of the parameter a for which this system is BIBO STABLE
(d) Find the impulse response of this system. Is the answer to part (c) EVIDENT FROM THE impulse response? Why?
(e) Let $a = 0.5$. Find the unit step response for this system.
(f) Let $a = 2.0$. Find the unit step response for this system.
(g) Check the results of parts (e) and (f) using MATLAB.

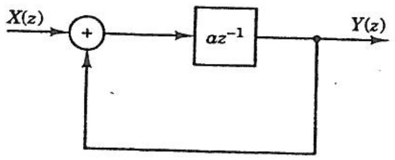

Figure P11.19

11.20. **(a)** Use the z-transform tables and the time-scaling property to find the inverse transforms of the following functions.

(i) $F(z) = \dfrac{z^3}{z^3 - 0.5}$ **(ii)** $F(z) = \dfrac{z^2}{z^3 - 0.5}$

(b) Give a sketch of each $f[n]$ in part (a).

(c) Verify the results in part (a) by a power-series expansion.

11.21. **(a)** Given a discrete-time function $f[n]$ and its z-transform $F(z)$. Suppose that $F(z)$ has a pole at $z = p_1$, where p_1 is real. Find the corresponding pole locations of the z-transform of $f[n/k]$, where k is a positive integer.

(b) In part (a), $F(z) = \mathscr{Z}[f[n]]$ is obtained by sampling $f(t)$ every T seconds. What is the sample period of $f[n/k]$?

(c) Sketch the pole locations found in part (a) for $p_1 = 0.9$ and $k = 3$.

(d) Repeat part (c) for the complex-pole pair $0.9 \angle \pm 30°$.

11.22. Consider a system with the transfer function $H(z)$.

(a) Give any third-order transfer function such that the system is causal but not stable.

(b) Give any third-order transfer function such that the system is not causal but stable.

(c) Give any third-order transfer function such that the system is neither causal nor stable.

(d) Give any third-order transfer function such that the system is both causal and stable.

11.23. **(a)** Determine the stability of the causal systems with the following transfer functions.

(i) $H(z) = \dfrac{3(z - 1.2)}{(z - 1)(z - 0.9)}$

(ii) $H(z) = \dfrac{3(z + 0.9)}{z(z - 0.9)(z - 1.2)}$

(iii) $H(z) = \dfrac{3(z - 0.9)}{z(z + 0.9)(z + 1.2)}$

(iv) $H(z) = \dfrac{3(z - 1)^2}{z^3 - 1.8z^2 + 0.81z}$

(v) $H(z) = \dfrac{2z - 1.5}{z^3 - 2z^2 + 0.99z}$

Use MATLAB where required.

(b) For each system that is unstable, give a bounded input for which the output is unbounded.

(c) Verify the results in part (b) by finding the unbounded term in the response for that input.

11.24. Given the general system transfer function

$$H(z) = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

Show that this system is causal provided that $a_0 \neq 0$. (*Hint*: Consider the impulse response.)

11.25. Find the bilateral z-transforms and the regions of convergence for the following functions.