UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

School of Electrical and Information Engineering

ELEN3013 Signals and Systems IIB

Tutorial 2

Ref 1: Signals, Systems and Transforms by Phillips and Parr, 2nd ed., 1999

1. Problem 10.3 from Ref 1.

2. For a system with impulse response

$$h(k) = 3u(k+1) - 3u(k-3)$$

and input

$$x(k) = 2u(k-1) - 2u(k-4)$$

where $u(k)$ is the unit step function, determine the system output $y(k)$ using convolution.

3. For a system with impulse response

$$h(k) = 5u(k-2) - 5u(k-9)$$

and input

$$x(k) = 2u(k-1) - 2u(k-6) + 10u(k-10) - 10u(k-16)$$

where $u(k)$ is the unit step function, determine the system output $y(k)$ using convolution.

4. Problem 10.9 from Ref 1.

5. Problem 10.11 from Ref 1.

6. Problem 10.12 from Ref 1.

7. Problem 10.14 from Ref 1.

8. Problem 10.15 from Ref 1.

9. Problem 10.17 from Ref 1.

10. Problem 10.20 from Ref 1.

11. Problem 10.21 from Ref 1.

12. A system is given by

$$y(k+2) + 0.5y(k+1) + 0.8y(k) = x(k+2) + 2x(k)$$

(a) Draw a block diagram of the system.

(b) Determine the impulse response of the system.

(c) Determine if the system is LTI.

(d) Determine if the system is causal.

(e) Determine if the system is BIBO stable.

(f) Determine the unit step response of the system.

As the final topic, the response of an LTI system to a sinusoidal input signal is derived. This derivation leads to the transfer-function description of an LTI system. It is shown in Chapter 11 that the transfer function allows us to find the response of an LTI system to any input signal. Hence the transfer function is also a complete input–output description of an LTI system.

REFERENCES

1. L. A. Pipes, *Applied Mathematics for Engineers*. New York: McGraw-Hill, 1946.

PROBLEMS

10.1. Given the convolution sum

$$y[n] = x[n]*h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Show that this sum can also be expressed as

$$y[n] = h[n]*x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

(*Hint:* Use a change of variables.)

10.2. Show that, for any function $g[n]$,

$$g[n]*\delta[n] = g[n]$$

10.3. **(a)** Evaluate $u[n]*u[n]$.
(b) Evaluate $u[n-n_1]*u[n-n_2]$, with $n_2 \geqq n_1 \geqq 0$.
(c) For $n = 5$, $n_1 = 1$, and $n_2 = 2$, sketch the convolution as in Figure 10.4 to verify the results in (b).

10.4 Given the LTI system of Figure P10.4, with the input $x[n]$ and the impulse response $h[n]$, where

$$x[n] = \begin{cases} 2, & 1 \leqq n \leqq 3 \\ 0, & \text{otherwise} \end{cases} \qquad h[n] = \begin{cases} 3, & -1 \leqq n \leqq 2 \\ 0, & \text{otherwise} \end{cases}$$

Parts (a), (b), and (c) are to be solved without finding $y[n]$ for all n.
(a) Solve for the system output at $n = 5$; that is, find $y[5]$.
(b) Find the maximum value for the output $y[n]$.
(c) Find the values of n for which the output is maximum.
(d) Verify the results by solving for $y[n]$ for all n.
(e) Verify the results of this problem using MATLAB.

Figure P10.4

10.5. Given the LTI system of Figure P10.4, with the impulse response $h[n] = \alpha^n u[n]$, where α is a constant. This system is excited with the input $x[n] = \beta^n u[n]$, with $\beta \neq \alpha$ and β constant.
(a) Find the system response $y[n]$. Express $y[n]$ in closed form, using the formulas for geometric series in Appendix C.

10.9. **(a)** Suppose that the discrete-time LTI system of Figure P10.4 has the impulse response $h[n]$ given in Figure P10.9(a). The system input is the unit step function $x[n] = u[n]$. Use the convolution sum to find the output $y[n]$.

(b) Repeat part (a) if the system input is $x[n]$ in Figure P10.9(b).

(c) Verify the results in part (b) using MATLAB.

(d) Repeat part (a) if the system input is $x[n]$ in Figure P10.9(c).

(e) Verify the results in part (d) using MATLAB.

(f) Repeat part (a) if $h[n]$ is the same function as $x[n]$ in Figure P10.9(b); that is, $h[n] = x[n]$.

(g) Verify the results in part (f) using MATLAB.

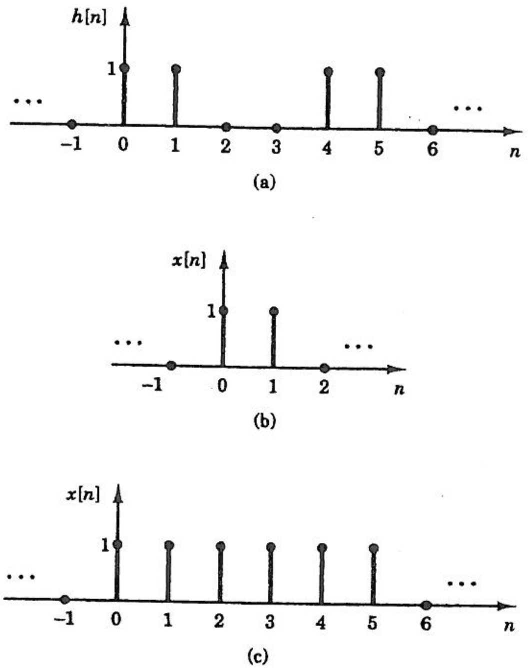

Figure P10.9

10.10. For the LTI system of Figure P10.4, the input signal is $x[n]$, the output signal is $y[n]$, and the impulse response is $h[n]$. For each of the cases below, use the convolution sum to find the output $y[n]$. The referenced signals are given in Figure P10.10.

(a) $x[n]$ in (a), $h[n]$ in (b)

(b) $x[n]$ in (a), $h[n]$ in (c)

(c) $x[n]$ in (a), $h[n]$ in (d)

(d) $x[n]$ in (b), $h[n]$ in (c)

(e) $x[n]$ in (b), $h[n]$ in (f)

(f) $x[n]$ in (α), $h[n]$ in (β), where α and β are assigned by your instructor

(g) Verify the results in each part using MATLAB.

10.11. For the system of Figure P10.4, suppose that $x[n]$ and $h[n]$ are identical and as shown in Figure P10.10(e).

(a) Find the output $y[n]$ for all n, by sketching $h[k]$ and $x[n - k]$.

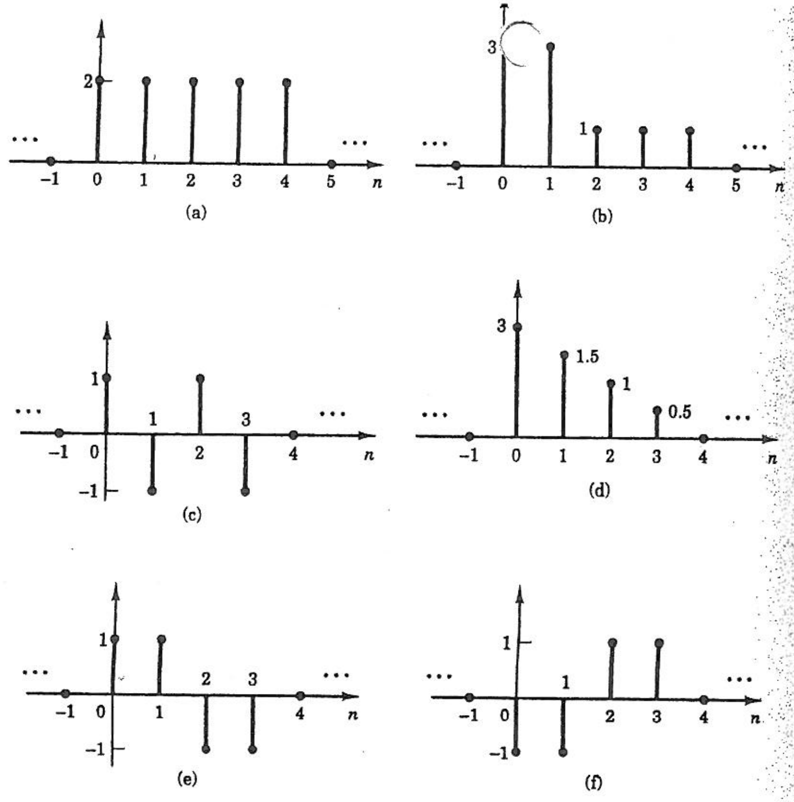

Figure P10.10

(b) Consider the expanded convolution sum of (10.15). Write out this expansion for each value of n in part (a), but include only those terms that are nonzero. Evaluate this expansion to verify the results of part (a).

(c) Verify the results using MATLAB.

10.12. **(a)** Consider the two LTI systems cascaded in Figure P10.12. The impulse responses of the two systems are identical, with $h_1[n] = h_2[n] = (0.8)^n u[n]$. Find the impulse response of the total system.

(b) Repeat part (a) for $h_1[n] = h_2[n] = \delta[n-3]$

(c) Write out the terms in part (b), as in (10.15), to verify the results.

(d) Repeat part (a) for $h_1[n] = h_2[n] = u[n] - [n-2]$. Express the results in terms of impulse functions.

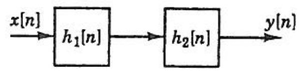

Figure P10.12

10.13. Show that the convolution of three signals can be performed in any order by showing that

(*Hint:* Form the required summations and use a change of variables)

10.14. An LTI discrete-time system has the impulse response

$$h[n] = (1.2)^n u[n]$$

(a) Determine if this system is causal.
(b) Determine if this system is stable.
(c) Find the system response to a unit step input $x[n] = u[n]$.
(d) Use MATLAB to verify the results in (c) for $n = 0, 1, 2,$ and 3.
(e) Repeat parts (a) through (c) for

$$h[n] = (1.2)^n u[-n]$$

(f) Repeat parts (a) and (b) for

$$h[n] = (0.9)^n u[-n]$$

(g) Repeat parts (a) and (b) for

$$h[n] = u[-n]$$

10.15. Consider an LTI system with the input and output related by

$$y[n] = 0.5(x[n+1] + x[n])$$

(a) Find the system impulse response $h[n]$.
(b) Is this system causal? Why?
(c) Determine the system response $y[n]$ for the input shown in Fig P10.15(a)
(d) Consider the interconnections of the LTI systems given in Fig 10.15(b) where $h[n]$ is the function found in part (a). Find the impulse response of the syst
(e) Solve for the response of the system of part (d) for the input of part (c)
(i) Using the results of part (c). This output can be written by inspectio
(ii) Using the results of part (d) and the convolution sum.

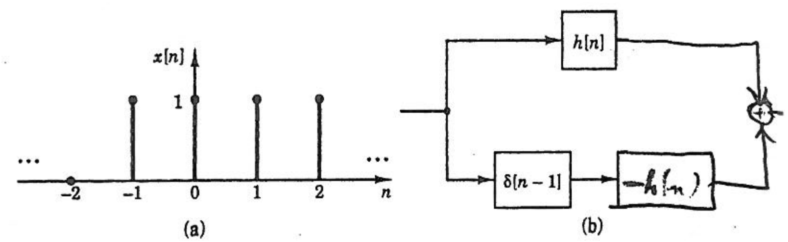

Figure P10.15

10.16. Consider a system described by the equation

$$y[n] = \cos(0.1\pi n)x[n]$$

(a) Is this system linear?
(b) Is this system time invariant?
(c) Determine the impulse response $h[n]$.
(d) Determine the response to the input $\delta[n-1]$.
(e) Can a linear time-varying system be described by its impulse

10.17. Determine the causality and the stability for the systems with the following impulse responses.

(a) $h[n] = e^{-3n}u[n]$
(b) $h[n] = e^{-3n}u[-n]$
(c) $h[n] = e^{3n}u[n]$
(d) $h[n] = \cos(3n)u[n]$
(e) $h[n] = ne^{-3n}u[n]$
(f) $h[n] = e^{-n}\cos(3n)u[n]$

10.18. **(a)** Given an LTI system with the output given by

$$y[n] = \sum_{k=0}^{n} e^{-2k}x[n-k]$$

Find the impulse response of this system.

(b) Is this system causal? Why?
(c) Is this system stable? Why?
(d) Repeat parts (a), (b), and (c) for an LTI system with the output given by

$$y[n] = \sum_{k=0}^{n} e^{-2(n-k)}x[k-1]$$

10.19. Suppose that the system of Figure P10.4 is described by each of the following system equations. Find the impulse response $h[n]$ for each of the systems.

(a) $y[n] = x[n-1]$
(b) $y[n] = \sum_{k=0}^{n} x[k]$

10.20 **(a)** Find the responses for systems described by the following difference equations with the initial conditions given.

(b) Verify that your response satisfies the initial conditions and the difference equation.

(i) $y[n] - 0.7y[n-1] = u[n], \quad y[-1] = -3$
(ii) $y[n] - 0.7y[n-1] = e^{-n}u[n], \quad y[-1] = 0$
(iii) $y[n] - 1.7y[n-1] + 0.72y[n-2] = u[n], \quad y[-2] = 1, \quad y[-1] = 0$
(iv) $y[n] - 0.7y[n-1] = \cos(n)u[n], \quad y[-1] = -1$

(c) Use MATLAB to verify your solutions in part (a) by finding $y(n)$ for $n = 0, 1, 2,$ and 3.

10.21. Consider a causal system with each of the system characteristic equations below.

(a) Give the modes of the system.
(b) Give the natural response for each of the systems.
If the characteristic equation has complex roots, express the response terms in the form of a sinusoid.

(i) $z - 0.9 = 0$
(ii) $z^2 - 1.7z + 0.72 = 0$
(iii) $z^2 + 2 = 0$
(iv) $z^3 - 1 = 0$
(v) $(z - 0.9)^3 = 0$
(vi) $(z - 0.9)(z - 1.2)(z + 0.85) = 0$

10.22 Determine the stability of each of the systems of Problem 10.21.

10.23 Given a discrete-time LTI system described by the difference equation

$$y[n] - 0.7y[n-1] = 2.5x[n] - x[n-1]$$

(a) Draw the form I realization (block diagram) for this system.
(b) Determine the impulse response $h[n]$, $0 \leq n \leq 4$, for the system.
(c) Verify the results of part (b) by tracing the impulse function through the block diagram of part (a).
(d) Suppose that the system input is given by