UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
School of Electrical and Information Engineering
ELEN3013 Signals and Systems IIB

Tutorial 1

Ref 1: Signals, Systems and Transforms by Phillips and Parr, 2nd ed., 1999

1. Problem 9.1 from Ref 1.

2. Problem 9.3 from Ref 1.

3. Problem 9.4 from Ref 1.

4. Problem 9.5 from Ref 1.

5. Problem 9.12 from Ref 1.

6. Problem 9.13 from Ref 1.

7. Problem 9.15 from Ref 1.

8. Problem 9.17 from Ref 1.

9. Problem 9.21 from Ref 1.

10. Problem 9.27 from Ref 1.

11. Problem 9.28 from Ref 1.

12. Problem 9.29 from Ref 1.

13. Problem 9.30 from Ref 1.

14. A signal

$$x(t) = A\cos(\pi t) + B\sin(2\pi t/3)$$

is ideally sampled at a sampling frequency 6π rad/sec to produce a discrete signal. Determine the minimum period for which the discrete signal is periodic.

mine the BIBO stability of LTI systems described by difference equations. No such statement can be made for other models. In addition, most digital-filter design procedures apply for LTI filters only.

SUMMARY

This chapter introduces discrete-time signals and systems. For a discrete-time signal $x[n]$, the discrete increment n represents time. First three transformations of the independent time variable n are defined: reversal, scaling, and shifting. Next the same three transformations are defined with respect to the amplitude of signals. A general procedure is developed for determining the effects of all six transformations. These transformations are important with respect to signals; they are equally important as transformations for functions of frequency. Frequency transformations will be covered when the discrete Fourier transformation is defined in Chapter 12.

The signal characteristics of evenness, oddness, and periodicity are defined next. These three characteristics appear often in the study of signals and systems.

Models of common signals that appear in certain types of physical systems are defined next. These signals include exponential signals and sinusoids whose amplitudes may vary exponentially. The impulse function is defined for discrete-time signals and is seen to be an ordinary function. It is shown that these discrete-time signals can be considered to be generated by sampling continuous-time signals. The study of periodic discrete signals is seen to be more complex than that of periodic continuous signals.

A general technique is given for expressing the output of a discrete-time system that is an interconnection of subsystems. As a final topic, some general properties of discrete-time systems are defined: memory, invertibility, causality, stability, time invariance, and linearity. For the remainder of this book, systems that are both linear and time invariant will be emphasized.

REFERENCES

1. C. L. Phillips and H. T. Nagle, *Digital Control System Analysis and Design*, 3rd ed. Upper Saddle River, NJ: Prentice Hall, 1996.

PROBLEMS

9.1. The discrete-time signals in Figure P9.1 are zero except as shown.

(a) For the signal $x_a[n]$ of Figure P9.1(a), plot:

(i)	$x_a[n/2]$	**(ii)**	$x_a[-2n]$
(iii)	$x_a[-n]$	**(iv)**	$x_a[3-n]$
(v)	$x_a[n-3]$	**(vi)**	$x_a[-1-n]$

(b) Repeat part (a) for the signal $x_b[n]$ of Figure P9.1(b).

(c) Repeat part (a) for the signal $x_c[n]$ of Figure P9.1(c).

(d) Repeat part (a) for the signal $x_d[n]$ of Figure P9.1(d).

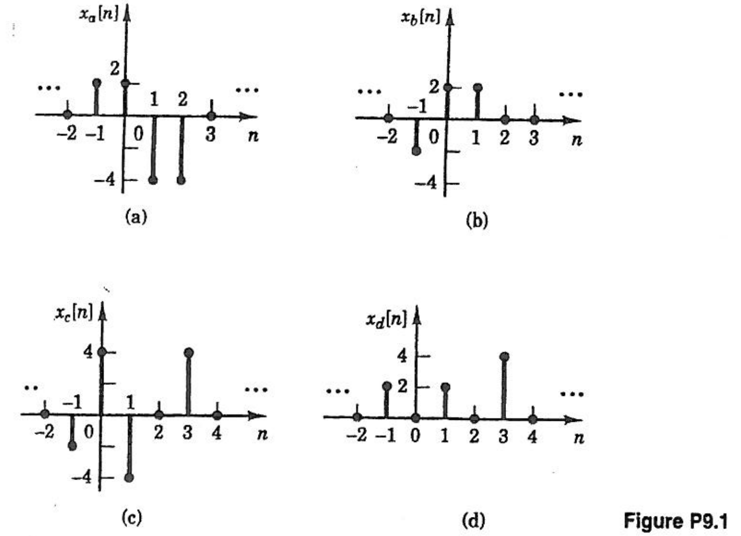

Figure P9.1

9.2. The signals in Figure P9.1 are zero except as shown.
(a) For the signal $x_a[n]$ of Figure P9.1(a), plot:
(i) $1 - 2x_a[n]$ **(ii)** $3x_a[-n]$
(iii) $-4x_a[n-3]$ **(iv)** $-2x_a[-n] + 2$
(v) $2 + 2x_a[-3+n]$ **(vi)** $4x_a[-n] - 2$
(b) Repeat part (a) for the signal $x_b[n]$ of Figure P9.1(b).
(c) Repeat part (a) for the signal $x_c[n]$ of Figure P9.1(c).
(d) Repeat part (a) for the signal $x_d[n]$ of Figure P9.1(d).

9.3. The signals in Figure P9.1 are zero except as shown.
(a) For the signal $x_a[n]$ of Figure P9.1(a), plot:
(i) $x_a[-n]u[n]$ **(ii)** $x_a[n]u[-n]$
(iii) $x_a[n]u[n-2]$ **(iv)** $x_a[n]u[2-n]$
(v) $x_a[n]\delta[n-1]$ **(vi)** $x_a[n](\delta[n] + \delta[n-2])$
(b) Repeat part (a) for the signal $x_b[n]$ of Figure P9.1(b).
(c) Repeat part (a) for the signal $x_c[n]$ of Figure P9.1(c).
(d) Repeat part (a) for the signal $x_d[n]$ of Figure P9.1(d).

9.4. Given the two signals in Figure P9.4:
(a) Express $x_1[n]$ as a function of $x_2[n]$.
(b) Verify your result by checking at least three points in time.

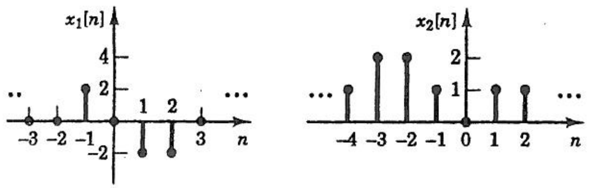

Figure P9.4

9.5. (a) For the general case of transformations of discrete signals, given $x[n]$, $x_t[n]$ can be expressed as

$$x_t[n] = Ax[an + n_0] + B$$

where a is rational and n_0 is an integer. Solve this expression for $x[n]$
(b) Suppose that for the signal of Figure P9.4,

$$x_1[n] = 0.5x_3[-n-1] + 2$$

Sketch $x_3[n]$.
(c) Verify the results of part (b) by checking at least three points in time

9.6. (a) Find the even and odd parts of $x_1[n]$ in Figure P9.4.
(b) Find the even and odd parts of $x_2[n]$.

9.7. (a) Plot the even and odd parts of the signal of Figure P9.1(a).
(b) Repeat part (a) for the signal of Figure P9.1(b).
(c) Repeat part (a) for the signal of Figure P9.1(c).
(d) Repeat part (a) for the signal of Figure P9.1(d).

9.8. (a) For each of the signals given, determine mathematically if the signal is even odd or neither.
(i) $x[n] = 2$ **(ii)** $x[n] = 2u[n]$
(iii) $x[n] = 2n$ **(iv)** $x[n] = 5 + 0.9^n + 0.9^{-n}$
(v) $x[n] = \cos(0.1n)$ **(vi)** $x[n] = \cos(0.1n - \pi/6)$
(b) Sketch the signals and verify the results of part (a).

9.9. Find the even part and the odd part of each of the signals given in Prol

9.10. (a) Given in Figure P9.10 are the *parts* of a signal $x[n]$ and its even par $n \geqq 0$. Note that $x_e[n] = 2$, $n \geqq 0$. Complete the plots of $x[n]$ and a plot of the odd part, $x_o[n]$, of the signal. Give the equations used f part of the signals.
(b) In Figure P9.10, let $x[0] = 0$, with all other values unchanged. Give this case for the results of part (a).

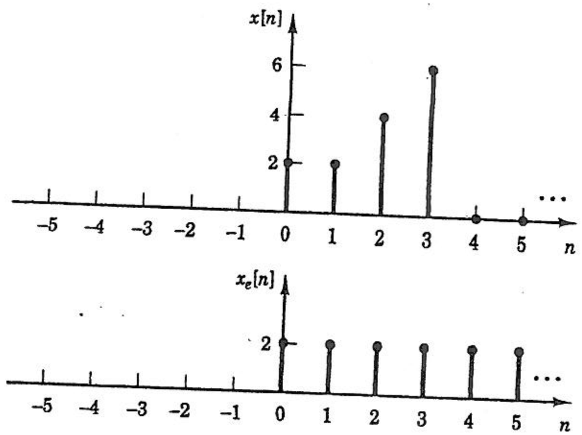

Figure P9.10

9.11. Let $x_e[n]$ and $x_o[n]$ be the even and odd parts, respectively, of $x[n]$.
(a) Show that $x_o[0] = 0$ and that $x_e[0] = x[0]$.
(b) Show that

$$\sum_{k=-\infty}^{\infty} x_o[n] = 0$$

(c) Show that

$$\sum_{k=-\infty}^{\infty} x[n] = \sum_{k=-\infty}^{\infty} x_e[n]$$

(d) Do the results of part (c) imply that

$$\sum_{k=n_1}^{n_2} x[n] = \sum_{k=n_1}^{n_2} x_e[n]$$

where n_1 and n_2 are any integers? Why?

9.12. Give proofs of the following statements.
(a) The sum of two even functions is even.
(b) The sum of two odd functions is odd.
(c) The sum of an even function and an odd function is neither even nor odd.
(d) The product of two even functions is even.
(e) The product of two odd functions is ~~odd~~ even.
(f) The product of an even function and an odd function is odd.

9.13. Suppose that the signals $x_1[n]$ and $x_2[n]$ are given by

$$x_1[n] = \cos(0.2\pi n), \quad x_2[n] = \cos(0.125\pi n)$$

(a) Determine if $x_1[n]$ is periodic. If so, determine the number of samples per fundamental period.
(b) Determine if $x_2[n]$ is periodic. If so, determine the number of samples per fundamental period.
(c) Determine if the sum of $x_1[n]$ and $x_2[n]$ is periodic. If so, determine the number of samples per fundamental period.

9.14. Given the discrete-time signals below. For each signal, determine the fundamental period N_0 if the signal is periodic; otherwise, prove that the signal is not periodic.
(a) $x[n] = e^{j5\pi n/7}$ **(b)** $x[n] = e^{j5n}$
(c) $x[n] = e^{j2\pi n}$ **(d)** $x[n] = e^{j0.3n/\pi}$
(e) $x[n] = \cos(3\pi n/7)$ **(f)** $x[n] = e^{j0.3n}$

9.15. A continuous-time signal $x(t) = \cos 2\pi t$ is sampled every T seconds, resulting in the discrete-time signal $x[n] = x(nT)$. Determine if the sampled signal is periodic for
(i) $T = 1\ s$ **(ii)** $T = 0.1\ s$
(iii) $T = 0.125\ s$ **(iv)** $T = 0.130\ s$
(v) $T = 5\ s$ **(vi)** $T = \frac{4}{3}\ s$
(b) For those sampled signals in part (a) that are periodic, find the number of periods of $x(t)$ in one period of $x[n]$.
(c) For those sampled signals in part (a) that are periodic, find the number of samples in one period of $x[n]$.

9.16. A continuous-time signal $x(t)$ is sampled at a 10-Hz rate, with the resulting discrete-time signals as given. Find the time constant τ for each signal, and the frequency ω of the sinusoidal signals.
(a) $x[n] = (0.7)^n$ **(b)** $x[n] = (0.5)^n \cos(3n)$
(c) $x[n] = (-0.7)^n$ **(d)** $x[n] = (0.1)^n \sin(0.2n + 1)$

9.17. (a) Determine which of the given signals are periodic.
(i) $x[n] = \cos(\pi n)$ **(ii)** $x[n] = -3\sin(0.01\pi n)$
(iii) $x[n] = \cos(3\pi n/2 + \pi)$ **(iv)** $x[n] = \sin(3.15n)$
(v) $x[n] = 1 + \cos(\pi n/2)$ **(vi)** $x[n] = \sin(3.15\pi n)$
(b) For those signals in part (a) that are periodic, determine the number of samples per period.

9.18. (a) In Problem 2.19 the time-scaling relation

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

was derived. Derive an equivalent time-scaling relation for the (function $\delta[n]$.
(b) Given the relation for continuous-time signals

$$u(t - t_0) = \int_{-\infty}^{\infty} \delta(t - t_0)dt$$

Derive an equivalent relation for discrete-time signals, expressing function of a summation of $\delta[m - N]$, where m is the summation is a given integer.
(c) Verify your result in part (b) by writing the first few terms of the $m = n, (n - 1), (n - 2), \ldots$. Then evaluate this summation for ? (i) $n = 3$ and (ii) $n = 0$.

9.19. (a) What is the difference between the unit step function $u[n + 4]$ and function $u[2n + 8]$.
(b) Repeat part (a) for $u[n]$ and $u[n/2]$.

9.20. Consider the signals shown in Figure P9.1.
(a) Write an expression for $x_a[n]$. The expression will involve the s impulse functions.
(b) Write an expression for $x_b[n]$.
(c) Write an expression for $x_c[n]$.
(d) Write an expression for $x_d[n]$.

9.21. Consider the signals shown in Figure P9.4.
(a) Write a mathematical expression for $x_1[n]$ in Figure P9.4. This EXPRESSION WILL involve the sum of discrete impulse functions.
(b) Express $x[n] = 1$ as infinite summation of impulse functions.
(c) Write a mathematical expression for $x_2[n]$ in Figure P9.4, using the RESULTS IN PART (b)
(d) Using the results of parts (a) and (b), show mathematically that

$$x_1[n] = 2 - 2x_2[-n - 1]$$

as shown in Problem 9.4.

9.22. The trapezoidal rule for numerical integration is defined in Figure P9 of the integral at $t = kT$ is equal to its value at $t = (k - 1)T$ plus tl area shown.
(a) Write a difference equation relating $y[k]$, the numerical integral o for this integrator.
(b) Write a MATLAB program that integrates $e^{-t}, 0 \leq t \leq 5$ s, with T trapezoidal integration.
(c) Run the program in part (b), and verify the result.

9.23. (a) Express the output $y(t)$ as a function of the input and the system tra in the form of (9.61), for the system of Figure P9.23.
(b) Repeat part (a), for the case that the summing junction with inputs y is replaced with a multiplication junction, such that its output is tl these two signals.

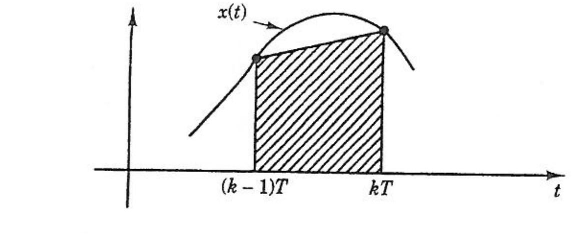

Figure P9.22

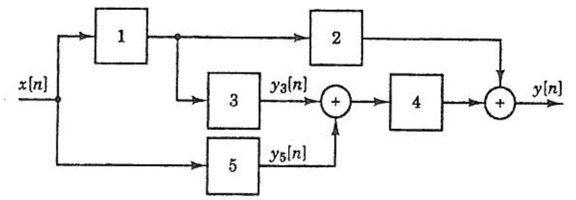

Figure P9.23

9.24. **(a)** Draw a block diagram, as in Figure 9.25, for a system described by

$$y_a[n] = T(x[n]) = T_2(x[n] + T_1(x[n])) + T_3(T_1(x[n]))$$

(b) Repeat part (a) for

$$y_b[n] = y_a[n] + x[n]$$

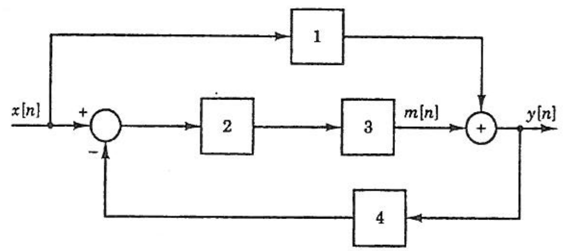

Figure P9.25

9.25. **(a)** Consider the feedback system of Figure P9.25. Express the output signal as a function of the transformation of the input signal, in the form of (9.61).
(b) Draw a block diagram, as in Figure P9.25, for a system described by

$$y[n] = T_2(T_1(x[n] - y[n]) - y[n])$$

9.26. For the system of Example 9.10, show that for $|x[n]| \leqq M$, $|y[n]| \leqq 9M$.

9.27. **(a)** Determine if the system described by

$$y[n] = \sin(x[n-1])$$

is

(i) Memoryless **(ii)** Invertible
(iii) Causal **(iv)** Stable
(v) Time invariant **(vi)** Linear

(b) Repeat part (a) for $y[n] = \ln x[n]$.
(c) Repeat part (a) for

$$y[n] = \frac{\sin x[n]}{x[n]}$$

Note that $\lim_{x\to 0} (\sin x)/x$ must be considered.
(d) Repeat part (a) for $y[n] = e^{x[n]}$.
(e) Repeat part (a) for $y[n] = e^{nx[n]}$.
(f) Repeat part (a) for $y[n] = 3x[n] + 2$.

9.28. The system described by the linear difference equation with consı

$$y[n] - 2y[n-1] + y[n-2] = x[n], \quad n \geqq$$

can be shown to be invertible and unstable. Determine if this systEM IS
(a) Memoryless **(b)** Causal
(c) Time invariant **(d)** Linear

9.29. **(a)** Determine if the summation operation, defined by

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

is
(i) Memoryless **(ii)** Invertible
(iii) Causal **(iv)** Stable
(v) Time invariant **(vi)** Linear

(b) Repeat part (a) for the averaging filter

$$y[n] = \tfrac{1}{2}[x[n] + x[n-1]]$$

(c) Repeat part (a) for the averaging filter

$$y[n] = \tfrac{1}{3}[x[n+1] + x[n] + x[n-1]]$$

9.30. **(a)** Sketch the characteristic y versus x for the system $y[n] = 2|x[n]|$. DETERMINE IF this system is:
(i) Memoryless **(ii)** Invertible
(iii) Causal **(iv)** Stable
(v) Time invariant **(vi)** Linear

(b) Repeat part (a) for

$$y[n] = \begin{cases} 3x[n], & x < 0 \\ 0, & x \geqq 0 \end{cases}$$

(c) Repeat part (a) for

$$y[n] = \begin{cases} -10, & x < -1 \\ 10x[n], & |x| \leqq 1 \\ 10, & x > 1 \end{cases}$$

(d) Repeat part (a) for

$$y[n] = \begin{cases} 2, & 2 < x \\ 1, & 1 < x \leqq 2 \\ 0, & 0 < x \leqq 1 \\ -1, & -1 < x \leqq 0 \\ -2, & x \leqq -1 \end{cases}$$

9.31. Let $h[n]$ denote the response of a system for which the input signal is the unit impulse function $\delta[n]$. Suppose that $h[n]$ for a *causal* system has the given *even* part $h_e[n]$ for $n \geqq 0$:

$$h_e[n] = \begin{cases} 0, & n = 0 \\ 1, & n = 1 \\ 2, & n \geqq 2 \end{cases}$$

Find $h[n]$ for all time, with your answer expressed as a mathematical function.