Lagrangian Modelling

(Double Pendulum Modelling)

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Double Pendulum

- Consider the Double Pendulum:

$$x_1 = l_1 sin\theta_1; \quad x_2 = l_1 sin\theta_1 + l_2 sin\theta_2; \tag{1}$$
$$y_1 = -l_1 cos\theta_1; \quad y_2 = -l_1 cos\theta_1 - l_2 cos\theta_2 \tag{2}$$

- kinetic(T) and potential energy(V), respectively are:

$$T = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{m_1(\dot{x}_1^2 + \dot{y}_1^2)}{2} + \frac{m_2(\dot{x}_2^2 + \dot{y}_2^2)}{2} \tag{3}$$
$$V = m_1 g y_1 + m_2 g y_2 \tag{4}$$

- Lagrangian,L:

$$\begin{aligned} L &= T - V = T_1 + T_2 - (V_1 + V_2) \\ &= \frac{m_1}{2}(\dot{x}_1^2 + \dot{y}_1^2) + \frac{m_2}{2}(\dot{x}_2^2 + \dot{y}_2^2) + m_1 g y_1 + m_2 g y_2 \end{aligned} \tag{5}$$

Double Pendulum p2

since

$$\dot{x}_1 = l_1 cos\theta_1\dot{\theta}_1; \quad \dot{x}_2 = l_1 cos\theta_1\dot{\theta}_1 + l_2 cos\theta_2\dot{\theta}_2; \tag{6}$$

$$\dot{y}_1 = l_1 sin\theta_1\dot{\theta}_1; \quad y_2 = l_1 sin\theta_1\dot{\theta}_1 + l_2 sin\theta_2\dot{\theta}_2 \tag{7}$$

T_1, T_2, V_1, and V_2 respectively are:
Lagrangian,L:

$$\begin{aligned} L &= T - V = T_1 + T_2 - (V_1 + V_2) \\ &= (\frac{m_1}{2} + \frac{m_2}{2})l_1^2\dot{\theta}_1^2 + \frac{m_2}{2}l_2^2\dot{\theta}_2^2 + m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2 cos(\theta_1 - \theta_2) \\ &+ (m_1 + m_2)gl_1 cos\theta_1 + m_2 gl_2 cos\theta_2 \end{aligned} \tag{8}$$

Double Pendulum p3

Lagrange equations a.k.a Euler-Lagrange equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = 0; \quad i = 1,2 \tag{9}$$

more generally

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} \quad = \quad u_i - \frac{\partial P}{\partial \dot{x}_i}; \quad i = 1,2,\ldots \tag{10}$$

LDC Machine Modelling 1

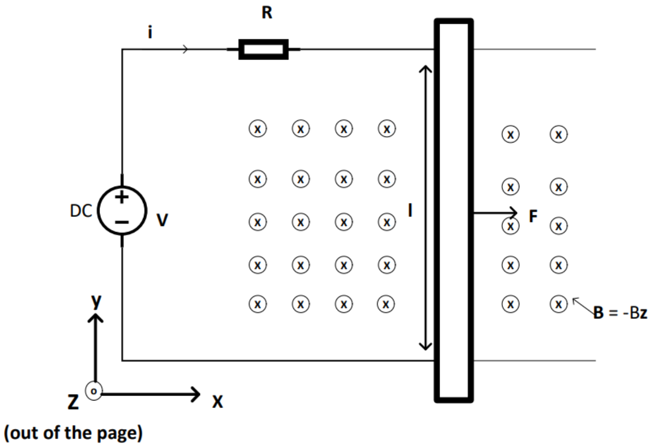

Figure: Chiasson linear DC machine [?]

LDC Machine Modelling 2

The bar experiences a Lorentz force,$\boldsymbol{F}$:

$$\boldsymbol{F} = i\boldsymbol{l} \times \boldsymbol{B} \tag{11}$$

For the linear DC machine shown in figure 1, the resultant force is expressed by:

$$\begin{aligned} \boldsymbol{F} &= il(-\hat{\boldsymbol{y}}) \times B(-\hat{\boldsymbol{z}}) \\ &= ilB\hat{\boldsymbol{x}} \end{aligned} \tag{12}$$

Using Kirchoff's voltage law, the current circuit i:

$$i = \frac{V - Bl\dot{x}}{R} \tag{13}$$

hence

$$m_l\ddot{x} - ilB = F_{app} - b\dot{x} \tag{14}$$

m_l is the bar mass, F_{app} is an external applied force and b is the bar to rail coefficient of rail friction.

LDC Machine Modelling 3

The Lagrange analysis steps can be summarized as follows:

- Identify the kinetic energy
- Identify the potential energy
- Identify the dissipation power
- Compute the Lagrange equation and input
- Evaluate the Euler-Lagrange
- Obtain the model

Project: System Modelling and Response Analysis

For the capacitive-microphone, single inverted pendulum, linear DC machine do the following:

- Physical Model
- Lagrange Model
- Simulink Model
- Simulation results for various input waveforms