Laplace, Fourier and z-transforms

Notation: T = sampling period $f_s = 1/T$

$E = e^{-\alpha T}$ $C = \cos(\beta T)$ $S = \sin(\beta T)$ $F = \alpha T - 1 + e^{-\alpha T}$ $G = 1 - e^{-\alpha T}(1 + \alpha T)$

$u(t)$ = unit step $\delta(t)$ = unit impulse

$\mathrm{rect}(\frac{t}{\tau})$ = rectangular pulse length τ $\mathrm{tri}(\frac{t}{\tau})$ = triangular pulse of length τ

$\mathrm{sinc}(x)$ = $\sin(\pi x)/\pi x$ $\delta_T(t)$ = unit impulse train of period T

$\mathrm{zoh}(\ldots)$ = zero-order hold acting on samples f_o = a particular frequency

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$x(t)$	$X(s)$	$X(f)$	$X(z)$
$x(t)$	$\int_0^\infty x(t)e^{-st}\,dt$	$\int_{-\infty}^\infty x(t)e^{-j2\pi ft}\,dt$	$\sum_{n=0}^\infty z^{-n}x(nT)$
$\int_{-\infty}^\infty X(f)e^{j2\pi ft}\,df$		$X(f)$	
$\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds$	$X(s)$		
$\delta(t)$	1	1	1
1		$\delta(f)$	
$\delta_T(t)$		$f_s\delta_{f_s}(f)$	
$u(t)$	$\frac{1}{s}$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$	$\frac{z}{z-1}$
$\mathrm{sgn}(t)$		$\frac{1}{j\pi f}$	
$tu(t)$	$\frac{1}{s^2}$		$\frac{Tz}{(z-1)^2}$
$\frac{t^2}{2}u(t)$	$\frac{1}{s^3}$		$\frac{T^2z(z+1)}{2(z-1)^3}$
$t^nu(t)$	$\frac{n!}{s^{n+1}}$		
$\mathrm{rect}(\frac{t}{\tau})$		$\tau\mathrm{sinc}\,(f\tau)$	
$\mathrm{sinc}\,(f_ot)$		$\frac{1}{f_o}\mathrm{rect}\left(\frac{f}{f_o}\right)$	
$\mathrm{tri}(\frac{t}{\tau})$		$\frac{\tau}{2}\ \mathrm{sinc}^2\left(\frac{f\tau}{2}\right)$	
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\frac{1}{\alpha+j2\pi f}$	$\frac{z}{z-E}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$	$\frac{1}{(\alpha+j2\pi f)^2}$	$\frac{TEz}{(z-E)^2}$
$t^2e^{-\alpha t}u(t)$	$\frac{2}{(s+\alpha)^3}$	$\frac{2}{(\alpha+j2\pi f)^3}$	$\frac{T^2Ez(z+E)}{(z-E)^3}$
$t^{n-1}e^{-\alpha t}u(t)$	$\frac{(n-1)!}{(s+\alpha)^n}$	$\frac{(n-1)!}{(\alpha+j2\pi f)^n}$	
$e^{-\alpha\|t\|}$		$\frac{2\alpha}{\alpha^2+4\pi^2f^2}$	
$e^{-\pi t^2}$		$e^{-\pi f^2}$	
$\frac{u(t)}{b-a}\left[e^{-at} - e^{-bt}\right]$	$\frac{1}{(s+a)(s+b)}$		
$\frac{u(t)}{b-a}\left[(c-a)e^{-at} - (c-b)e^{-bt}\right]$	$\frac{s+c}{(s+a)(s+b)}$		

$x(t)$	$X(s)$	$X(f)$	$X(z)$
$(1-e^{-\alpha t})u(t)$	$\frac{\alpha}{s(s+\alpha)}$		$\frac{z(1-E)}{(z-1)(z-E)}$
$(\alpha t-1+e^{-\alpha t})u(t)$	$\frac{\alpha^2}{s^2(s+\alpha)}$		$\frac{z(Fz+G)}{(z-1)^2(z-E)}$
$e^{\pm j2\pi f_o t}$		$\delta(f \mp f_o)$	
$\cos(2\pi f_o t)$		$0.5[\delta(f-f_o)+\delta(f+f_o)]$	
$\sin(2\pi f_o t)$		$-j0.5[\delta(f-f_o)-\delta(f+f_o)]$	
$u(t)e^{-\alpha t}\cos(\beta t)$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{\alpha+j2\pi f}{\alpha^2+\beta^2+j4\pi\alpha f-4\pi^2 f^2}$	$\frac{z^2-ECz}{z^2-2ECz+E^2}$
$u(t)e^{-\alpha t}\sin(\beta t)$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{\beta}{\alpha^2+\beta^2+j4\pi\alpha f-4\pi^2 f^2}$	$\frac{ESz}{z^2-2ECz+E^2}$
$x(t)\delta_T(t)$		$\frac{1}{T}\sum_{n=-\infty}^{\infty} X(f-nf_s)$	
$\mathrm{zoh}(x(nT))$	$\frac{1-e^{-sT}}{s}X(s)$	$\frac{2}{T}\mathrm{sinc}\,(fT)\,e^{-j\pi fT}$	$\frac{z-1}{z}\mathcal{Z}\left(\frac{X(s)}{s}\right)$
$x(t-\tau)$	$e^{-s\tau}X(s)$	$e^{-j2\pi f\tau}X(f)$	
$x(t-nT)$			$z^{-n}X(z)$
$x(t)e^{j2\pi f_o t}$		$X(f-f_o)$	
$x(t)e^{s_o t}$	$X(s-s_o)$		
$x(t)\cos 2\pi f_o t$		$\frac{1}{2}\{X(f-f_o)+X(f+f_o)\}$	
$x(t)\sin 2\pi f_o t$		$-\frac{j}{2}\{X(f-f_o)-X(f+f_o)\}$	
$x(at)$	$\frac{1}{\lvert a\rvert}X(\frac{s}{a})$	$\frac{1}{\lvert a\rvert}X(\frac{f}{a})$	
$\frac{1}{\lvert b\rvert}x(\frac{t}{b})$	$X(bs)$	$X(bf)$	
$x^*(t)$		$X^*(-f)$	
$\int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau$	$X(s)Y(s)$	$X(f)Y(f)$	
$x(t)y(t)$		$\int_{-\infty}^{\infty} X(u)Y(f-u)\,du$	
$X(t)$		$x(-f)$	
$\frac{dx}{dt}$	$sX(s)-x(0_+)$	$j2\pi fX(f)$	
$\frac{d^n x}{dt^n}$	$s^nX(s)-\sum_{k=0}^{n-1} s^{n-k-1}\frac{d^k x}{dt^k}(0)$	$(j2\pi f)^n X(f)$	
$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	$\frac{1}{j2\pi f}X(f)+\frac{X(0)}{2}\delta(f)$	
$\frac{x(t_-)+x(t_+)}{2}$		$X(f)$	
$\int_{-\infty}^{\infty} x^*(t)y(t)\,dt$		$\int_{-\infty}^{\infty} X(f)Y^*(f)\,df$	