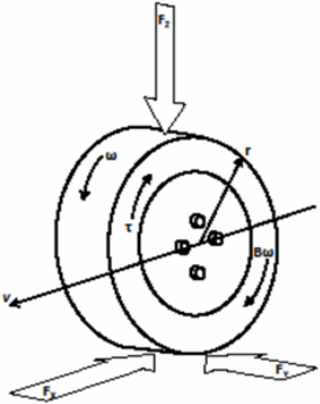
Supplementary Notes:Accelleration & Braking of Wheeled Vehicles, Active suspension and Thermometers

ABS MODEL FORMULATION

Formulation of the continuous time model for control purposes control consists of: ABS model formulation

- physical model via force and torque analysis
- state model formulation
- state model analysis

ABS physical model



- analyse forces
- analyse torques
- model based on force and torque equations
- analyse reletive magnitudes of forces
- analyse reletive magnitudes of torques
- derive state model
- perform state change(transform) if necessary

ABS model formulation continued

The dynamic braking equations for the vehicle derived from Newton's laws:

$$\dot{v} = -\mu(\lambda)F_z/M - C_x v^2/M \quad (1)$$

$$\dot{\omega} = \mu(\lambda)F_z r/I - B\omega/I - \tau_b/I \quad (2)$$

$$\mu(\lambda) = \frac{2\mu_0\lambda_0\lambda}{\lambda_0^2 + \lambda^2} \quad (3)$$

$$\lambda_{braking} = \frac{v - r\omega}{v} \quad or \quad \lambda_{accelleration} = \frac{r\omega - v}{v} \quad (4)$$

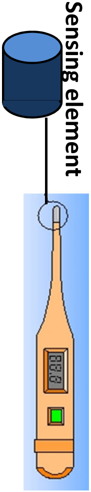
- The state variables are: $x_1 = \omega$, $x_2 = v$
- The control input is $u = \tau_b$.
- The controlled output is $y = \lambda$.

THERMOMETER MODEL FORMULATION

Formulation of the continuous time model for control purposes control consists of: ABS model formulation

- physical model via energy transfer
- state model formulation
- state model analysis

Thermometer energy transfer model



Analyse energy transfer *via*:

- change in heat content of sensing element
- heat transfer by convection
- heat transfer by conduction
- heat transfer by radiation
- other heat losses
- analyse relative magnitudes of each heat transfer element
- derive state model
- perform state change(transform) if necessary

Thermometer model formulation continued

Principle of conservation of (heat) energy analysis :

$$\dot{Q} = hA(T_f - T) \quad \textit{convective heat transfer} \quad (5)$$

$$\dot{U} = mc_p \frac{dT}{dt} \quad \textit{internal energy increase rate} \quad (6)$$

$$hA(T_f - T) = mc_p \frac{dT}{dt}; \quad \textit{note} \quad \tau = \frac{mc_p}{hA} \quad (7)$$

- The state variable: $x = T$; the control input is $u = T_f$.
- T is the instantaneous sensing element temperature (^oC)
- T_f is the fluid temperature being monitored (^oC)
- m is mass of sensing element (kg); c_p is specific heat capacity (W/(kg ^oC)); h is the convective heat transfer coefficient (W/(m^{2o}C))

Thermometer model formulation continued

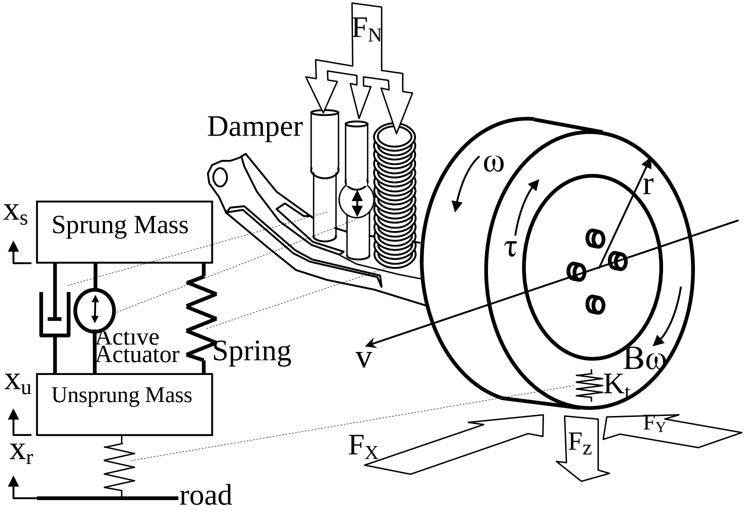
Principle of conservation of (heat) energy analysis :

$$\tau \dot{T} + T = T_f \quad or \ \dot{T} + \frac{T}{\tau} = \frac{T_f}{\tau} \tag{8}$$

$$T(t) = T_f + (T_0 - T_f)e^{-t/\tau} \tag{9}$$

- Use La Place and time domain based analysis to determine (9) from (8)
- note $\tau = mc_p/hA$ determines transient response of thermometer
- faster thermometer response is from smaller m, smaller c_p, higher A

System Diagram



Active suspension Equations

The suspension dynamic equations are derived from Newton's laws:

$$m_s \ddot{x}_s = -k_s(x_s - x_u) - b_s(\dot{x}_s - \dot{x}_u) + u_s$$
$$m_u \ddot{x}_u = k_s(x_s - x_u) + b_s(\dot{x}_s - \dot{x}_u) - k_w x_u - u_s \quad (10)$$

a note on springs and dampers:

$$F_s = k_s(x) + k_s^{nl}(x)^3 \quad (11)$$
$$F_d = b_s(\dot{x}) + b_s^{nl}(\dot{x}) + b_s^{sym}\left|(\dot{x}_s - \dot{x}_u)\right| \quad (12)$$

- The control input is the active suspension force $u = u_s$
- is the model (10) linear
- **Develop the above model as a Tutorial/homework exercise**