CONTROL I

ELEN3016

Closed-Loop Control Systems

(Lecture 8)

Overview

- First Things First!
- Closed-Loop Control Systems
- Examples
- Tutorial Exercises & Homework

- Next Attraction!

Proportional Control

1st-Order System

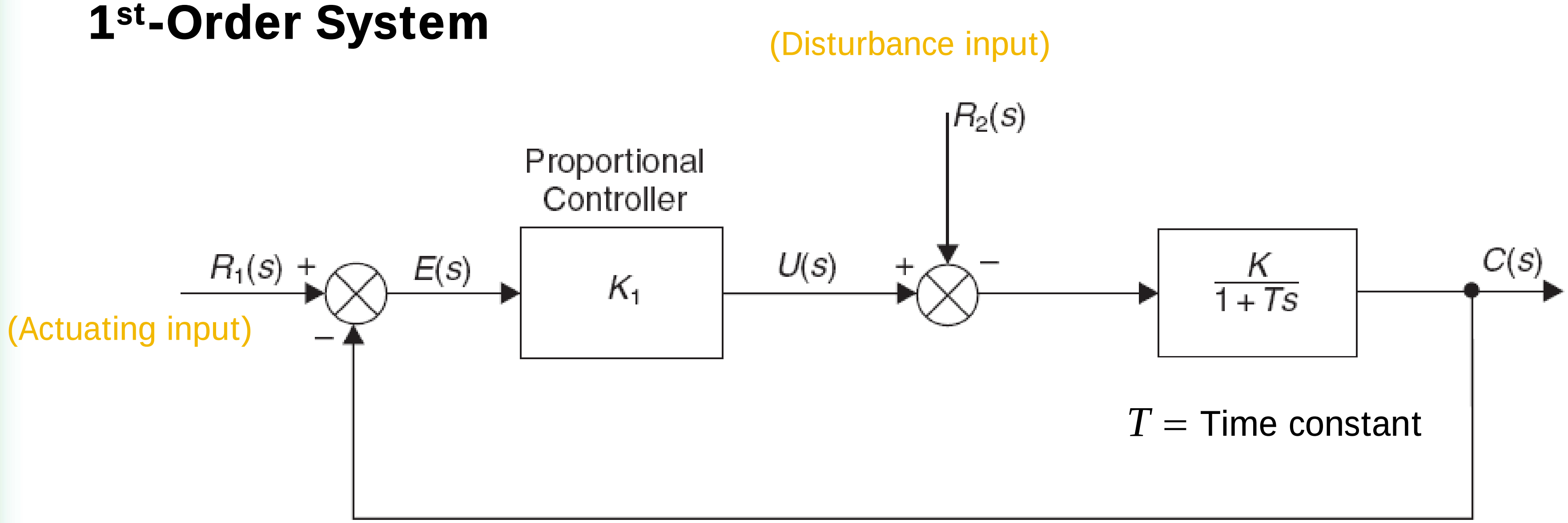

Fig. 4.23 Proportional control of a first-order plant.

$$u(t) = K_1\, e(t) \qquad\qquad U(s) = K_1\, E(s)$$

Proportional Control

Superposition yields

$$C(s) = \frac{\left(\frac{K_1 K}{1+K_1 K}\right) R_1(s) - \left(\frac{K}{1+K_1 K}\right) R_2(s)}{\left\{1 + \left(\frac{T}{1+K_1 K}\right) s\right\}} \quad (4.64)$$

FVT

$$c(t)\Big|_{\substack{r_1(t)=u(t) \\ r_2(t)=0}} = \left(\frac{K_1 K}{1 + K_1 K}\right) \quad \text{as } t \to \infty.$$

$$c(t)\Big|_{\substack{r_1(t)=0 \\ r_2(t)=u(t)}} = -\left(\frac{K}{1 + K_1 K}\right) \quad \text{as } t \to \infty.$$

Proportional Control

Ideally we require that

$$\left(\frac{K_1 K}{1 + K_1 K}\right) = 1$$
$$\left(\frac{K}{1 + K_1 K}\right) = 0 \qquad (4.65)$$

requiring that $K_1 K = \infty$ (physically impossible!). Practically all we can do is to set $K_1 K$ to be a large as is possible, yielding

$$\left(\frac{K_1 K}{1 + K_1 K}\right) \approx 1$$
$$\left(\frac{K}{1 + K_1 K}\right) \approx 0$$

Proportional Control

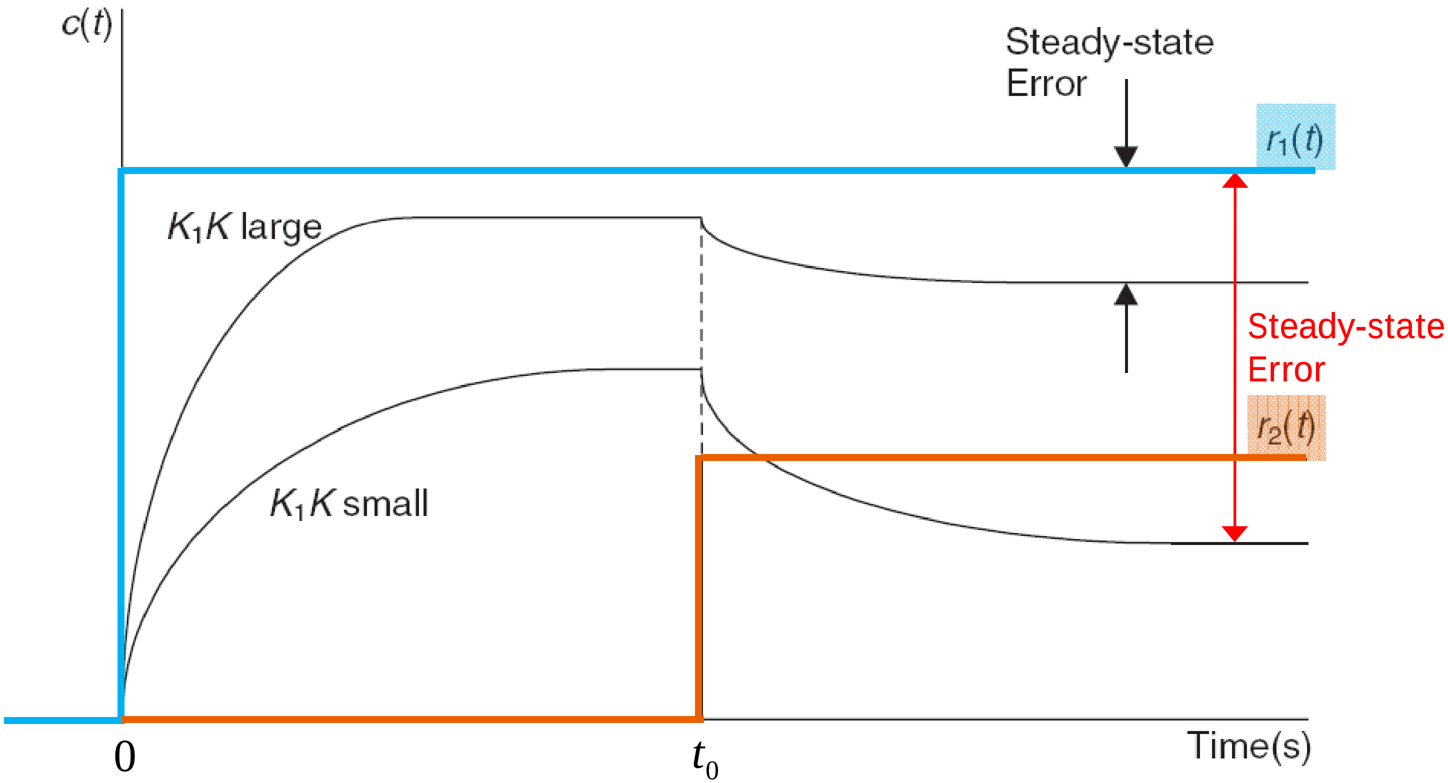

Fig. 4.24 Step response of a first-order plant using proportional control.

Notice: The non-zero steady-state error.

Proportional + Integral Control

1st-Order System

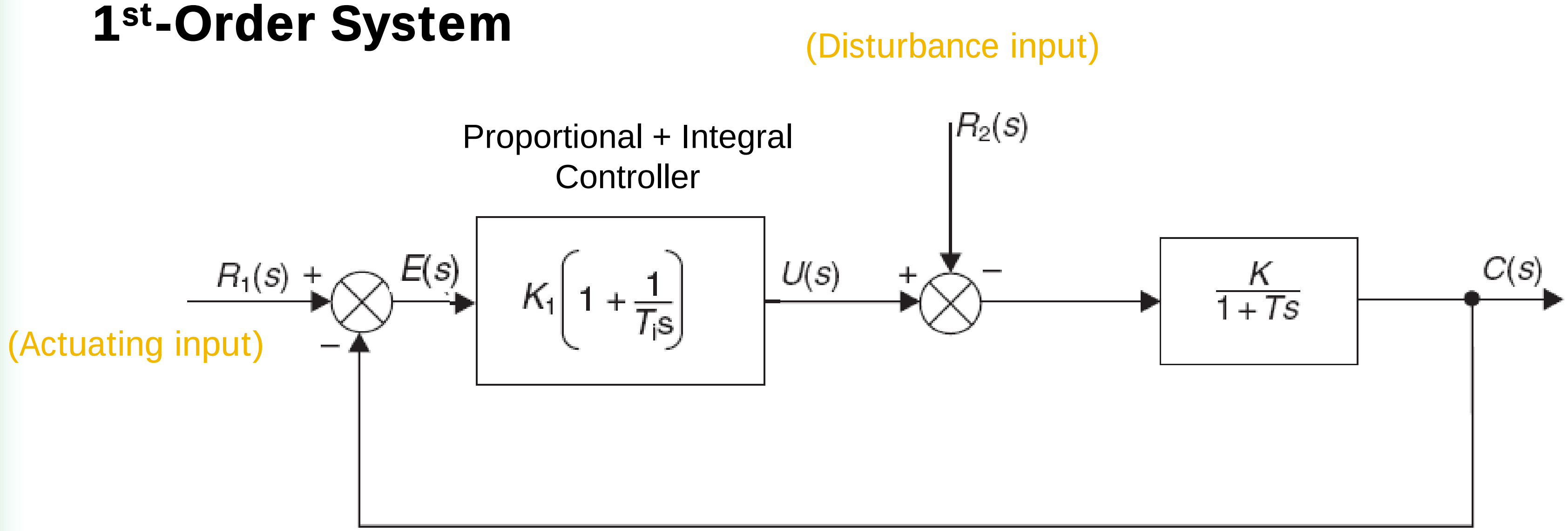

Figure. Proportional + Integral control of the first-order plant.

Open-loop poles: $s = 0, \qquad s = -\frac{1}{T}$

Proportional + Integral Control

$$u(t) = K_1 e(t) + K_2 \int e \, \mathrm{d}t \tag{4.67}$$

$$\begin{aligned} U(s) &= \left(K_1 + \frac{K_2}{s}\right) E(s) \\ &= K_1 \left(1 + \frac{K_2}{K_1 s}\right) E(s) \\ &= K_1 \left(1 + \frac{1}{T_\mathrm{i} s}\right) E(s) \end{aligned} \tag{4.68}$$

Proportional + Integral Control

Superposition yields

$$C(s) = \frac{(1 + T_i s)R_1(s) - T_i s R_2(s)/K_1}{\underbrace{\left(\frac{T_i T}{K_1 K}\right)}_{\omega_n^{-2}} s^2 + \underbrace{T_i\left(1 + \frac{1}{K_1 K}\right)}_{2\zeta\omega_n^{-1}} s + 1} \qquad (4.71)$$

FVT

$$c(t)\Big|_{t\to\infty} = \lim_{s\to 0} sC(s)$$

$$= r_1(t)\Big|_{t\to\infty} - 0 \cdot r_2(t)\Big|_{t\to\infty}$$

$$= r_1(t)\Big|_{t\to\infty}$$

Proportional + Integral Control

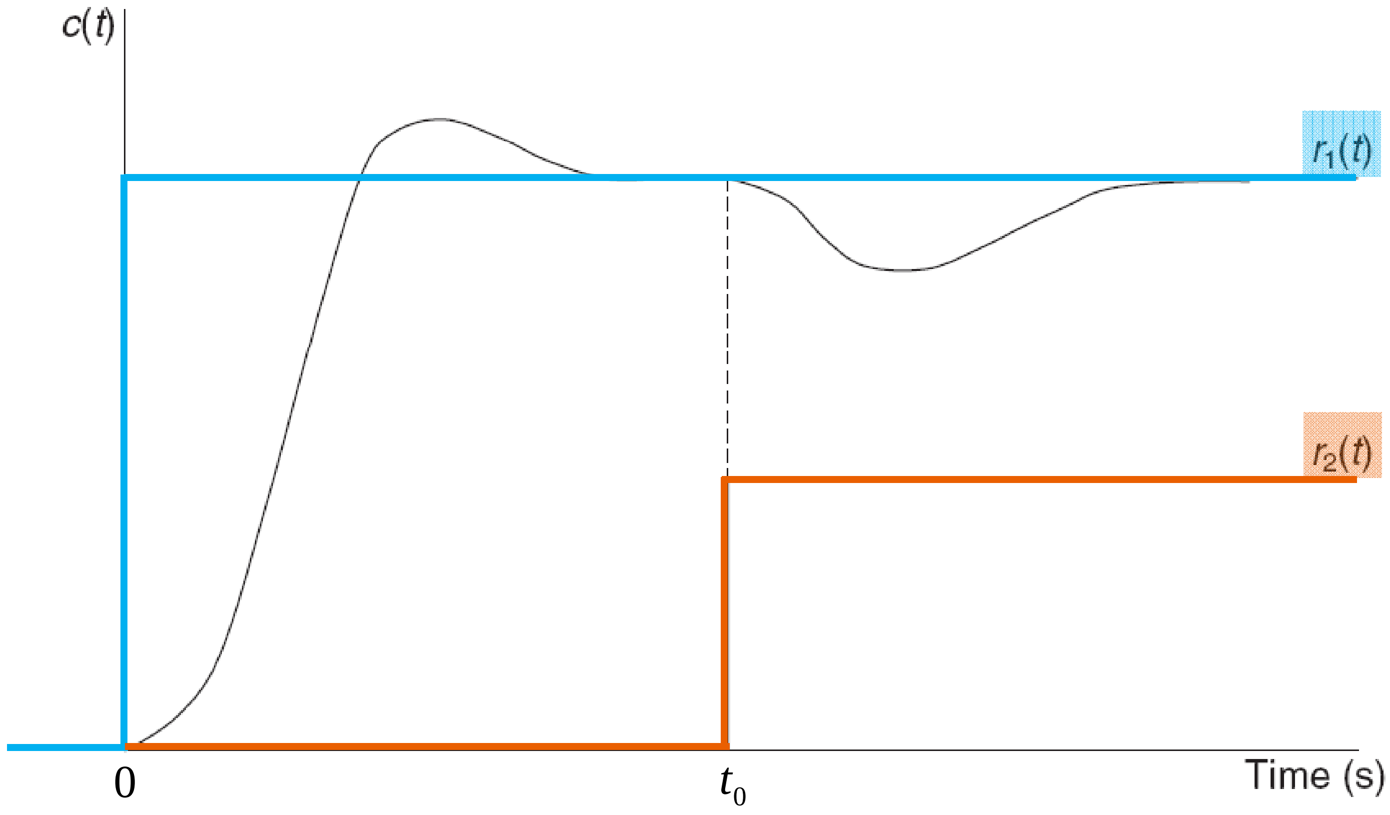

Fig. 4.25 Step response of a first-order plant using PI control.

PID Control

$$u(t) = K_1 e(t) + K_2 \int e\,\mathrm{d}t + K_3 \frac{\mathrm{d}e}{\mathrm{d}t} \tag{4.90}$$

Taking Laplace transforms, yields

$$\begin{aligned} U(s) &= \left(K_1 + \frac{K_2}{s} + K_3 s \right) E(s) \\ &= K_1 \left(1 + \frac{K_2}{K_1 s} + \frac{K_3}{K_1} s \right) E(s) \\ &= K_1 \left(1 + \frac{1}{T_\mathrm{i} s} + T_\mathrm{d} s \right) E(s) \end{aligned} \tag{4.91}$$

$$U(s) = \frac{K_1 (T_\mathrm{i} T_\mathrm{d} s^2 + T_\mathrm{i} s + 1)}{T_\mathrm{i} s} E(s) \tag{4.92}$$

PID Control

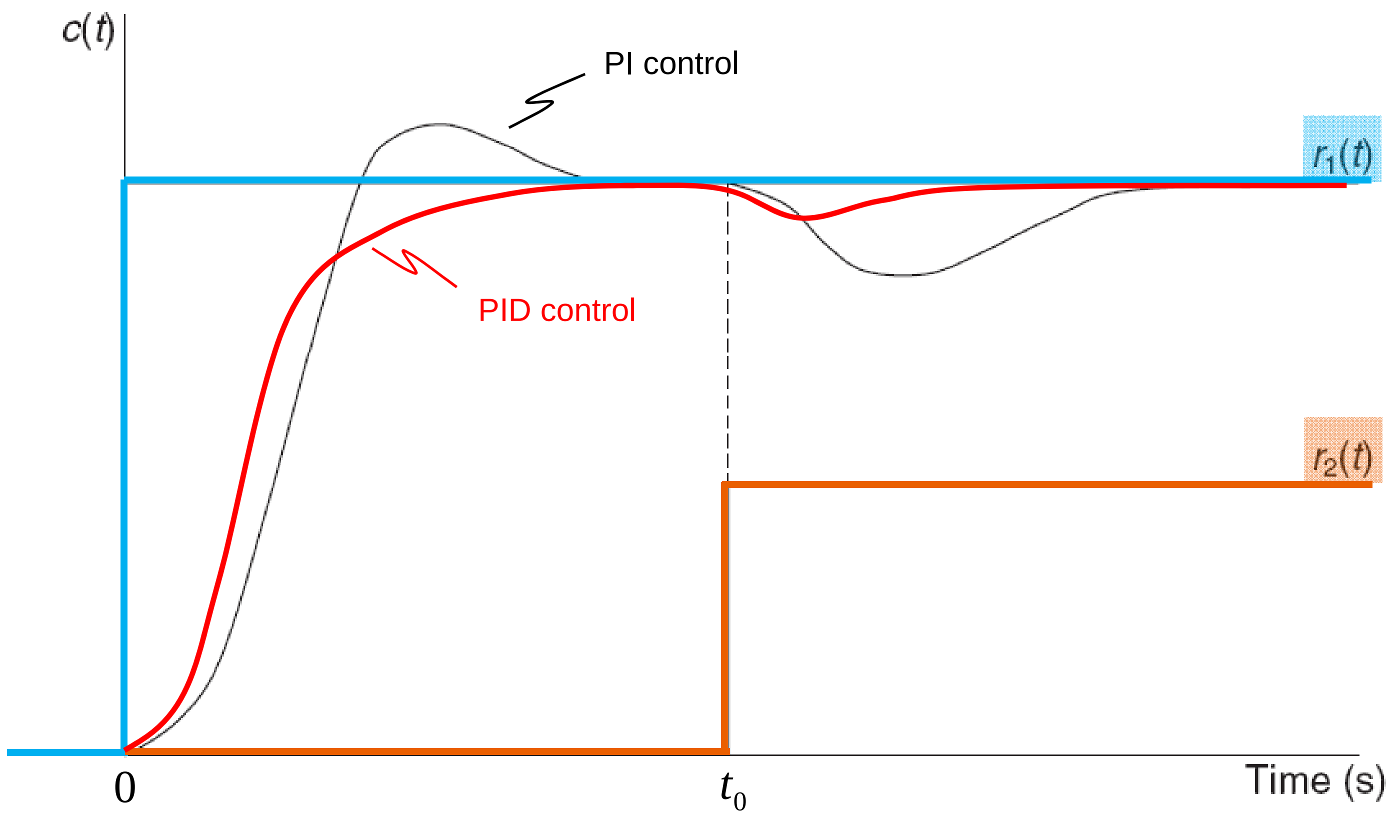

Figure. Proportional + Integral + Derivative control of the first-order plant.

PID Control

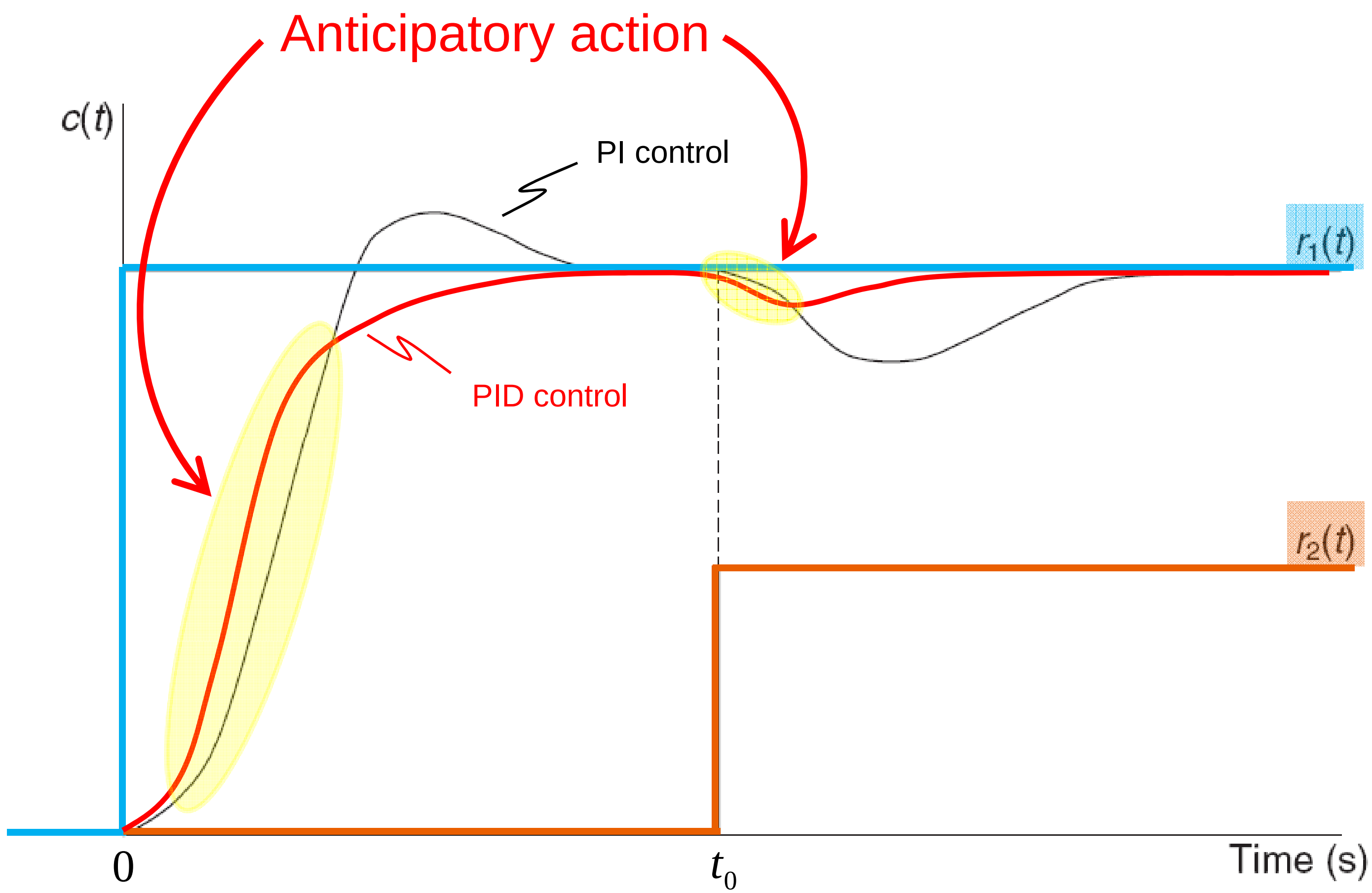

Figure. Proportional + Integral + Derivative control of the first-order plant.

Tutorial Exercises & Homework

- Tutorial Exercise
  - Example 4.9
- Homework
  - Burns, Example 4.5

Conclusion

- Closed-Loop P-control of 1st-order systems
- Closed-Loop PI-control of 1st-order systems
- Closed-Loop PID-control
- Example 4.5 (Self-study!)
- Tutorial Exercises & Homework

Next Attraction! – Miss It & You'll Miss Out!

- Ziegler-Nichols Tuning Method
- Case Study

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Thank you!

Any Questions?