

CONTROL I

ELEN3016

Discrete Signals & Systems and Z-Transform

(Lecture 22)

Overview

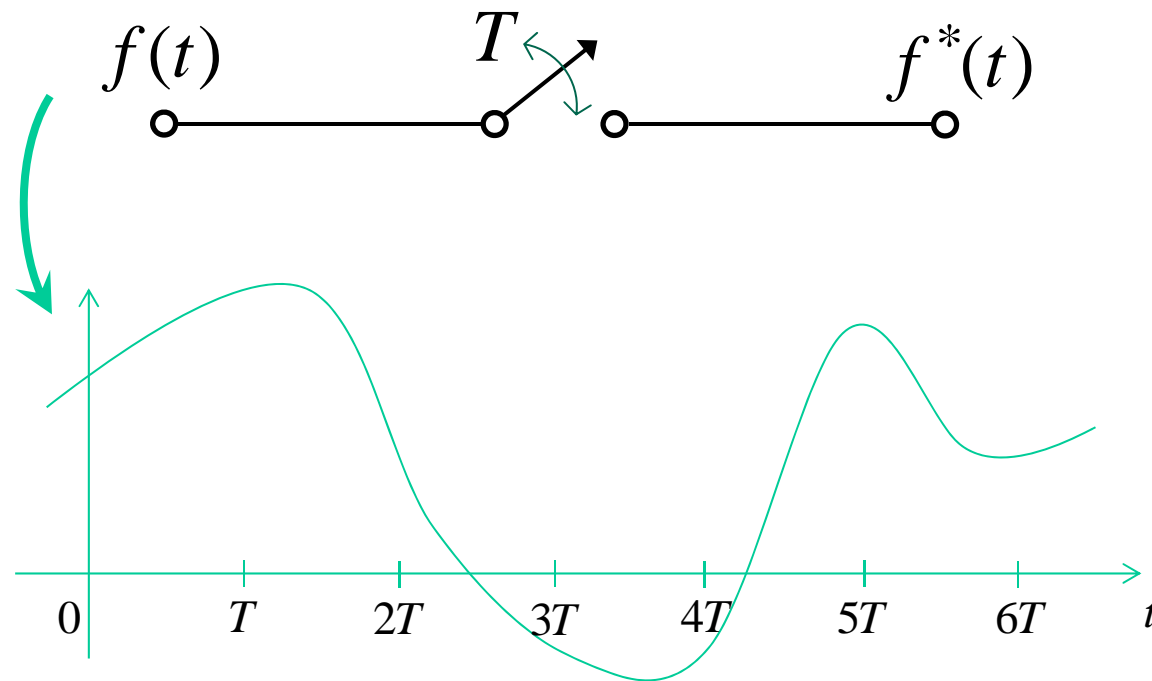
- First Things First!
- Signals and Sampling
- Z-Transforms
- Systems and Sampling
- Digital Controller Design
(Classical Methods, Numerical Methods, Analytical Design, Optimum Response Digital Design)
- Tutorial Exercises & Homework

First Things First!

- Tut on Thursday 20 September 2012
- Lab assessment booking sheets are available at reception.

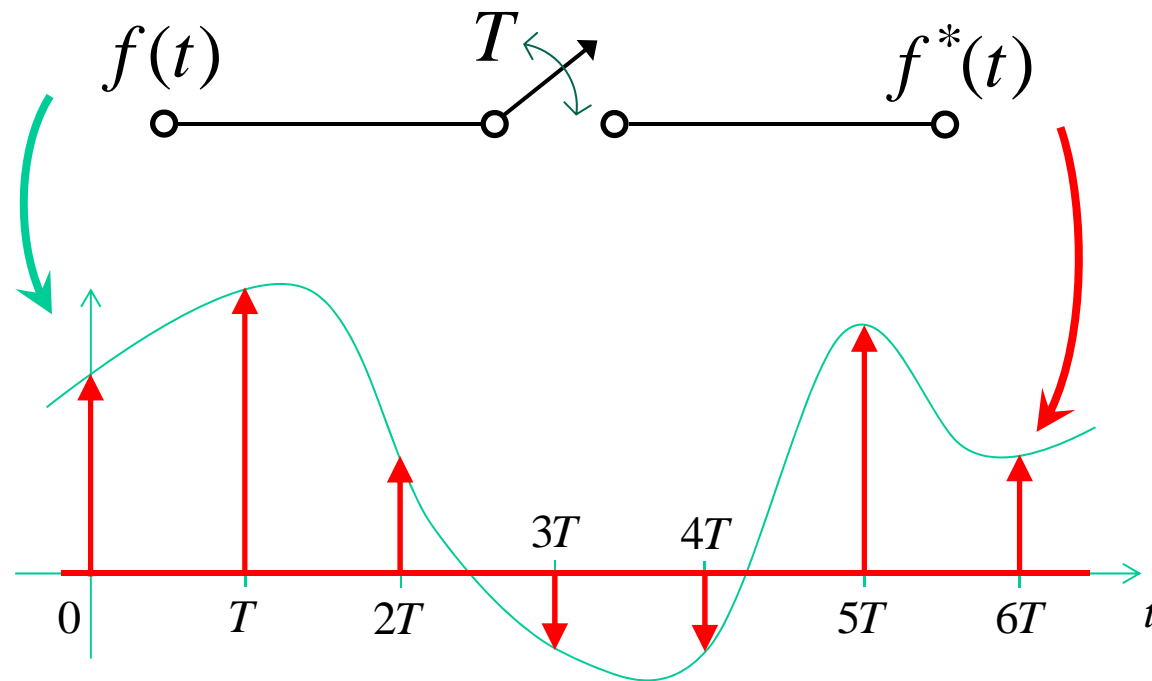
Signals and Sampling

- Signal Sampling



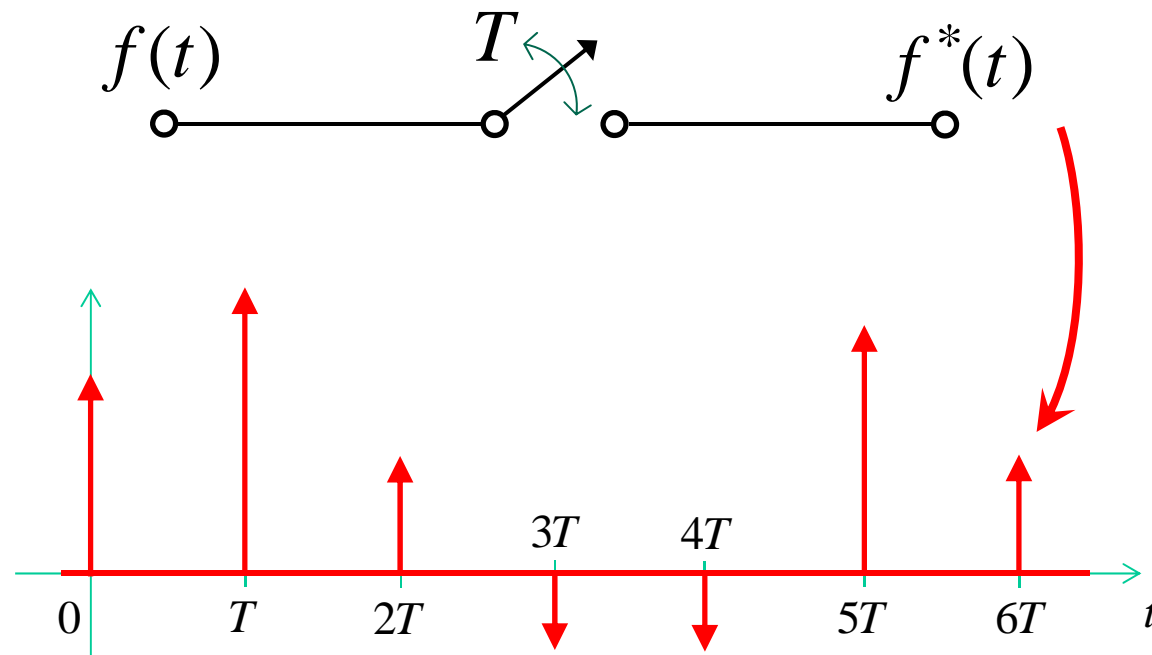
Signals and Sampling

- Signal Sampling cont'd



Signals and Sampling

- Signal Sampling cont'd



Signals and Sampling

- Signal Sampling cont'd

Sampled signal as a convolution sum

$$f^*(t) = \sum_{n=0}^{\infty} f(nT)\delta(t - nT)$$

Taking the Laplace transform yields

$$F^*(s) = \mathcal{L}(f^*(t)) = \sum_{n=0}^{\infty} f(nT)e^{-snT}$$

Z-Transform: $F(z) = \mathcal{Z}(f(t)) = \mathcal{L}(f^*(t))_{e^{sT} \mapsto z}$

Z-Transform

- Some Important Z-Transforms

$$1. \quad Z[a^n] = \sum_{k=0}^{\infty} a^k z^{-k} = \sum_{k=0}^{\infty} \left(\frac{a}{z}\right)^k = \frac{1}{1 - \frac{a}{z}} = \frac{z}{z - a}$$

$$2. \quad Z[na^n] = Z\left[a \frac{d}{da} (a^n)\right] = a \frac{d}{da} Z[a^n] = \frac{az}{(z - a)^2}$$

$$3. \quad Z[n] = Z\left[na^n \Big|_{a=1}\right] = Z[na^n]_{a=1} = \frac{z}{(z - 1)^2}$$

$$4. \quad Z[e^{j\omega n}] = Z\left[a^n \Big|_{a=e^{j\omega}}\right] = Z[a^n]_{a=e^{j\omega}} = \frac{z}{z - e^{j\omega}}$$

Z-Transform

- Some Important Z-Transforms cont'd

$$5. \quad Z[\sin \omega n] = \frac{1}{2j} Z[e^{j\omega n} - e^{-j\omega n}] = \dots = \frac{z \sin \omega}{z^2 - 2z \cos \omega + 1}$$

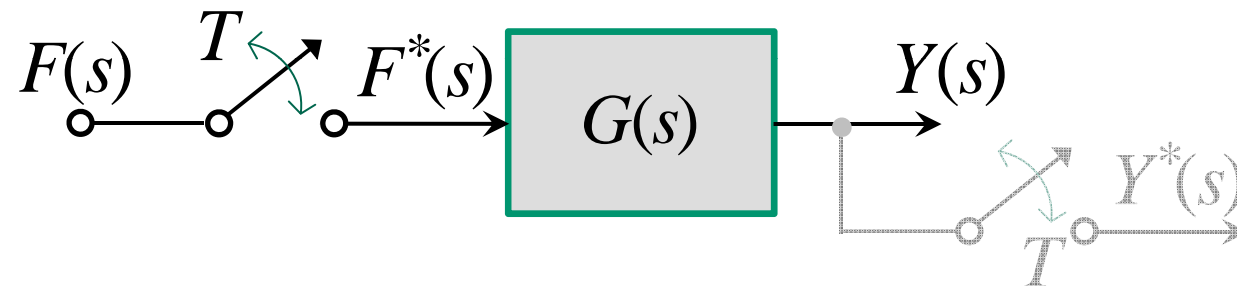
$$6. \quad Z[\delta(n)] = Z\left[\lim_{a \rightarrow 0} a^n\right] = \lim_{a \rightarrow 0} Z[a^n] = 1$$

$$7. \quad Z[u(n)] = Z\left[a^n \Big|_{a=1}\right] = Z[a^n]_{a=1} = \frac{z}{z-1}$$

Conclusion: Many transforms follow from that of the standard exponential function in 1.) above.

Systems and Sampling

- Block Diagram Algebra – Example 1



For the above system we have

$$y(t) = \sum_{k=0}^{\infty} f(kT)g(t - kT) \quad (\text{Signals \& Systems})$$

and

$$y^*(t) = \sum_{n=0}^{\infty} y(nT)\delta(t - nT)$$

Systems and Sampling

- Block Diagram Algebra – Example 1 cont'd

$$y^*(t) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} f(kT)g(nT - kT)\delta(t - nT)$$

Taking the Laplace transform yields

$$\begin{aligned} Y^*(s) &= \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} f(kT)g(nT - kT)e^{-snT} \\ &= \sum_{k=0}^{\infty} f(kT) \sum_{n=0}^{\infty} g(nT - kT)e^{-snT} \end{aligned}$$

Systems and Sampling

- Block Diagram Algebra – Example 1 cont'd

$$\begin{aligned} Y^*(s) &= \sum_{k=0}^{\infty} f(kT) G^*(s) e^{-skT} \\ &= G^*(s) \left(\sum_{k=0}^{\infty} f(kT) e^{-skT} \right) \\ &= G^*(s) F^*(s) \end{aligned}$$

Letting $e^{sT} \mapsto z$ yields $Y(z) = G(z) F(z)$.

Systems and Sampling

- Block Diagram Algebra cont'd

From the above we obtain the following algebraic rules:

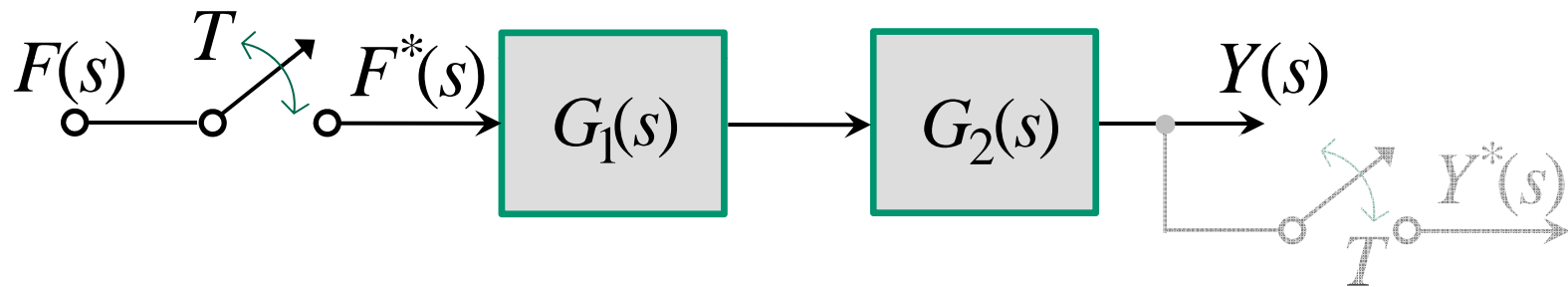
1. $[G^*(s)]^* = G^*(s)$

2. $\mathcal{L}(y(t))^* = \mathcal{L}(y^*(t))$ i.e. $[Y(s)]^* = Y^*(s)$

3. $[F^*(s) G(s)]^* = F^*(s)[G(s)]^* = F^*(s) G^*(s)$

Systems and Sampling

- Block Diagram Algebra – Example 2



From the above system we have

$$Y(s) = G_1(s)G_2(s)F^*(s).$$

Applying the star-operator yields

$$Y^*(s) = \left[G_1(s)G_2(s)F^*(s) \right]^* = \left[G_1(s)G_2(s) \right]^* F^*(s).$$

Systems and Sampling

- Block Diagram Algebra – Example 2 cont'd

For ease of reference we define

$$G_1 G_2^*(s) := [G_1(s) G_2(s)]^*$$

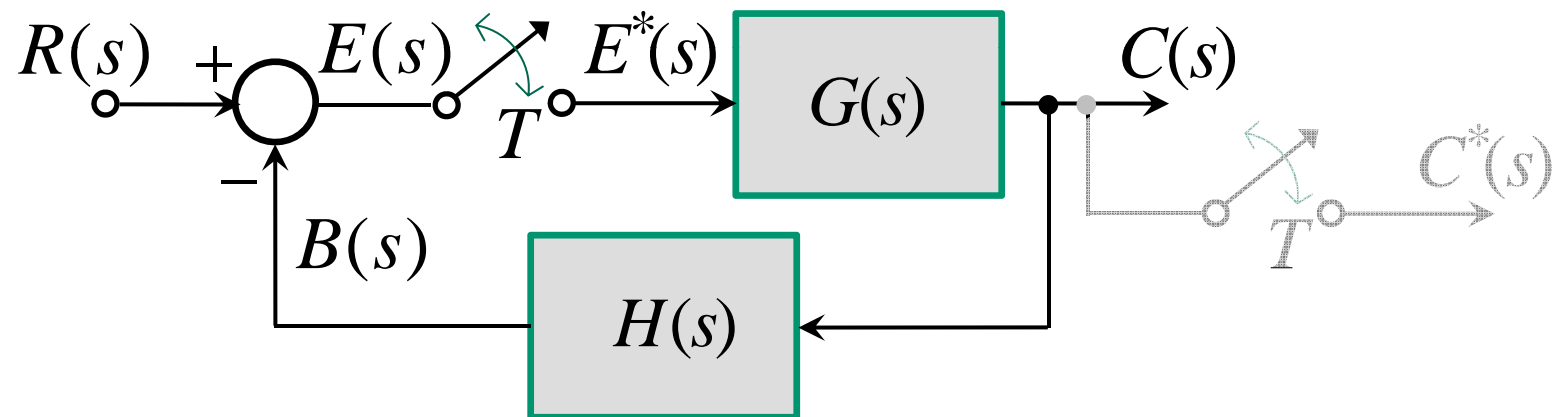
hence

$$Y^*(s) = [G_1 G_2(s)]^* F^*(s)$$

Letting $e^{sT} \mapsto z$ yields $Y(z) = G_1 G_2(z) F(z)$.

Systems and Sampling

- Block Diagram Algebra – Example 3



For the above system we have

$$C(s) = G(s) E^*(s) \quad \Rightarrow \quad C^*(s) = G^*(s) E^*(s). \quad (1)$$

Systems and Sampling

- Block Diagram Algebra – Example 3 cont'd

Furthermore

$$\begin{aligned} E(s) &= R(s) - B(s) \\ &= R(s) - H(s)C(s) \\ &= R(s) - G(s)H(s)E^*(s). \end{aligned}$$

Applying the star-operator yields

$$E^*(s) = R^*(s) - GH^*(s)E^*(s).$$

Systems and Sampling

- Block Diagram Algebra – Example 3 cont'd

Solving for $E^*(s)$ gives

$$E^*(s) = \frac{R^*(s)}{1 + GH^*(s)}. \quad (2)$$

Substituting (2) into (1) yields

$$C^*(s) = \frac{G^*(s)}{1 + GH^*(s)} R^*(s).$$

Systems and Sampling

- Block Diagram Algebra – Example 3 cont'd

Letting $e^{sT} \mapsto z$ yields

$$\begin{aligned} C(z) &= C^*(s) \Big|_{e^{sT} \mapsto z} \\ &= \left[\frac{G^*(s)}{1 + GH^*(s)} R^*(s) \right]_{e^{sT} \mapsto z} \\ &= \frac{G(z)}{1 + GH(z)} R(z). \end{aligned}$$

Tutorial Exercises & Homework

- Tutorial Exercises

- Prove that $G_1 G_2(z) \neq G_1(z) G_2(z)$ in general.

- Homework

- None

Conclusion

- Z-transforms
- Signals & Sampling
- Tutorial Exercises & Homework

Next Attraction! – Miss It & You'll Miss Out!

- Digital Control System Design
(Burns, Chapter 7)

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**Thank you
for your interest!**