CONTROL I

ELEN3016

Classical Design in the Frequency Domain

(Lecture 21)

Overview

- First Things First!
- Phase-Lag Compensator Design Methods
- Phase-Lag Compensator Example
- Tutorial Exercises & Homework

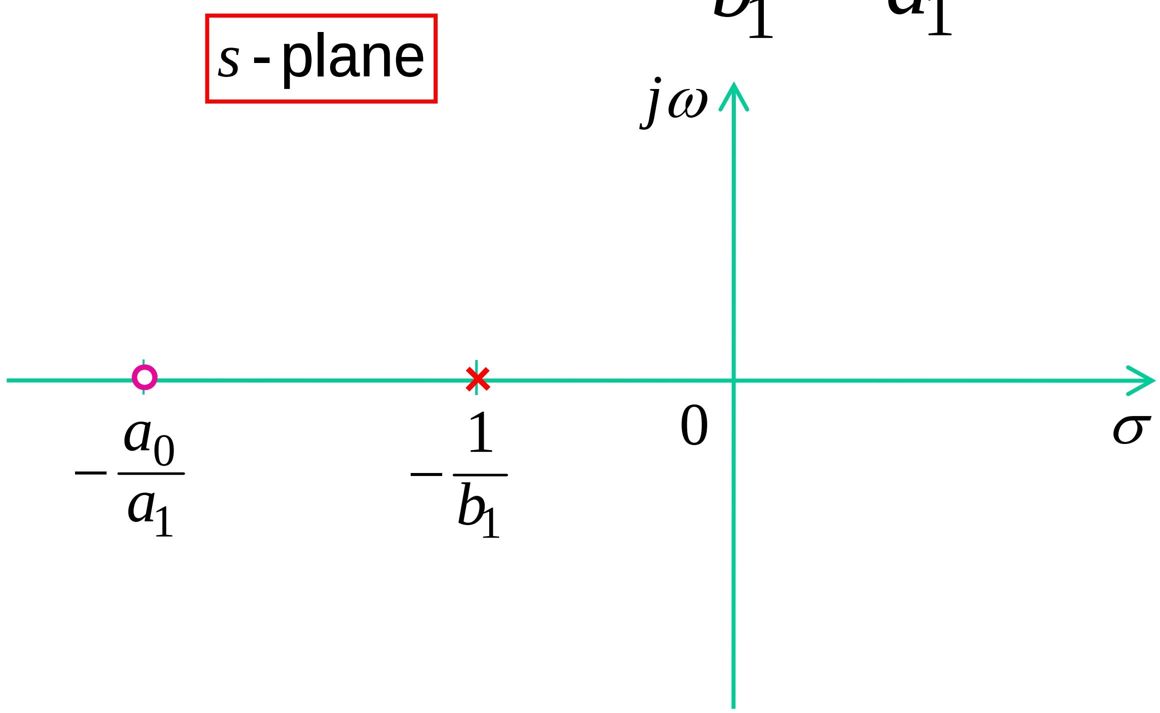
- Next Attraction!

First Things First!

- None

1^{st}-Order Controllers

- Approximate PI Controller
  - Requirement: $p = \dfrac{1}{b_1} < \dfrac{a_0}{a_1} = z$



1st-Order Controllers

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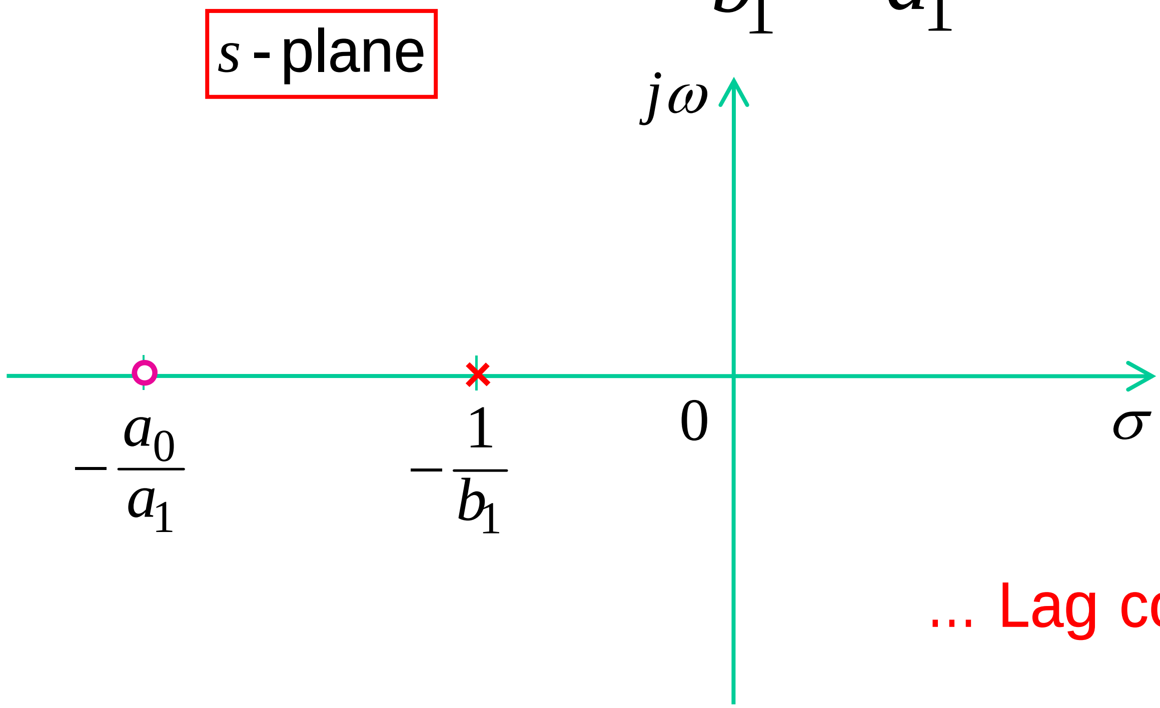
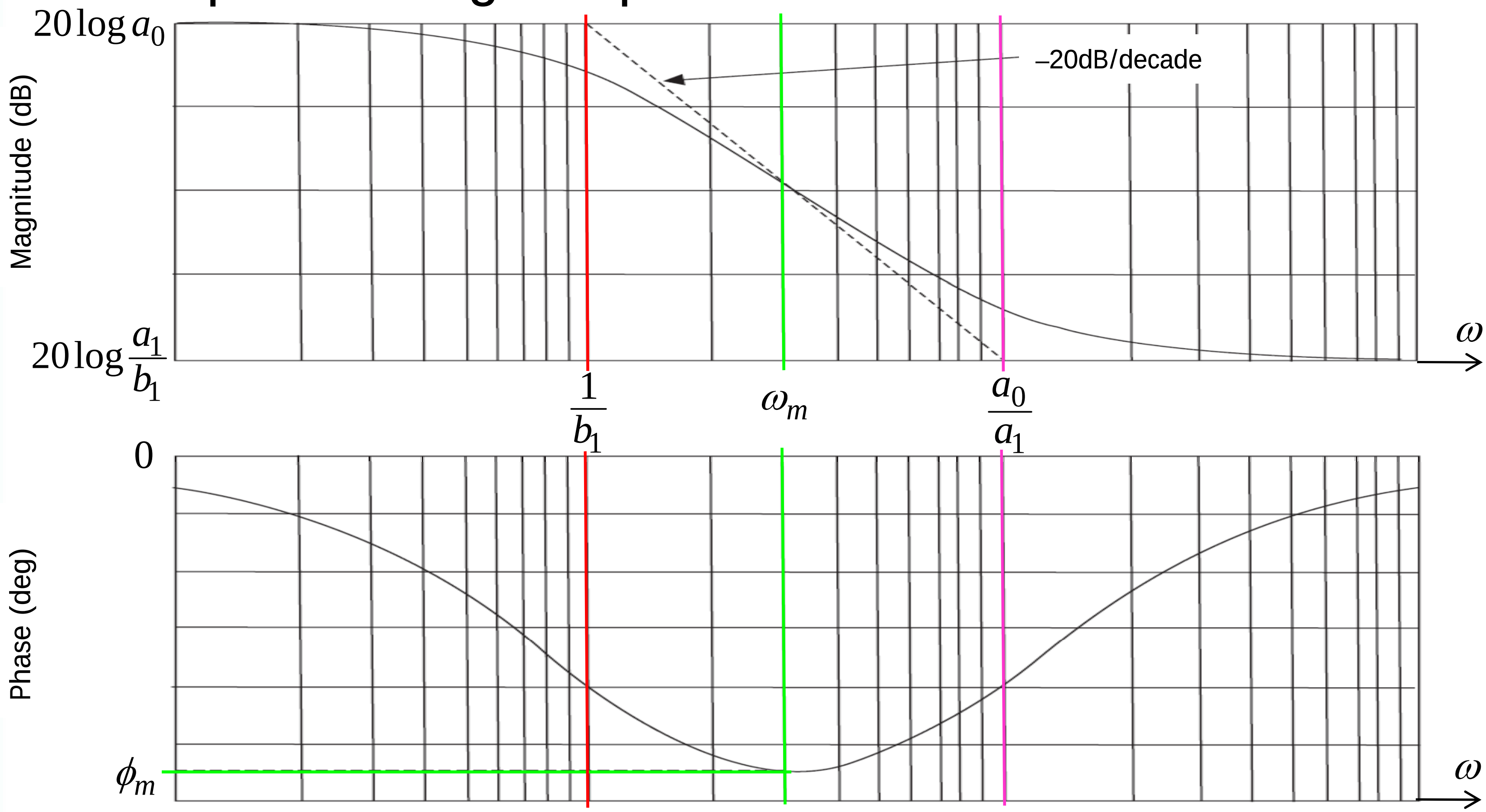


... Lag compensator

Lag Compensator

Bode plots for lag compensator



Lag Compensator

- Analysis of the Phase-Lag Compensator
  - Frequency at maximum phase lag is

$$\omega_m = \sqrt{\frac{a_0}{a_1 b_1}}$$

  - Corresponding maximum phase lag is

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha} \quad \text{or} \quad \alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

Lag Compensator

- Lag Compensator Design – Method 1
  - Select a_0 to meet any specified steady-state error spec.
  - Plot the uncompensated open-loop frequency response. From it determine the magnitude value, say $|G_p(j\omega_0)|$, associated with the phase $5° + PM - 180°$ and set $1/\alpha = |G_p(j\omega_0)|$. When the lag compensator is combined with the plant this point will become the unity gain point of the frequency resp. The corresponding frequency will be $10a_0/a_1$, so $\omega_0 = 10a_0/a_1$.
  - From these two equations we can now solve for a_1 and b_1.

Lag Compensator

- Method 1 cont'd
  - Draw compensated open-loop Bode plots and inspect design.
  - Close the loop and determine appropriate closed-loop responses (i.e. transient response and frequency response).

Study the examples in Burns and in Raven.

Lag Compensator

- Lag Compensator Design – Method 2
  - Select a_0 to meet any specified steady-state error spec.
  - Plot the frequency response of $a_0 G_p(j\omega)$.
  - Given the modulus-crossover frequency ω_{gc} as well as phase margin PM required, the compensated open-loop system must satisfy

$$G_c(j\omega_{gc})G_p(j\omega_{gc}) = \frac{a_1 j\omega_{gc} + a_0}{b_1 j\omega_{gc} + 1} \underbrace{M_G e^{j\theta_G}}_{G_p(j\omega_{gc})} = 1e^{j(-180^\circ + PM)}$$

giving the following design equations,

$$a_1 = \frac{1 + a_0 M_G \cos(PM - \theta_G)}{-\omega_{gc} M_G \sin(PM - \theta_G)} \qquad b_1 = \frac{\cos(PM - \theta_G) + a_0 M_G}{\omega_{gc} \sin(PM - \theta_G)}$$

Lag Compensator

- Method 2 cont'd
  - Draw compensated open-loop Bode plots and inspect design.
  - Close the loop and determine appropriate closed-loop responses (i.e. transient response and frequency response).

Lag Compensator

- Example
  - Plant, $G_p(s) = \dfrac{10}{s(s+5)}$.
  - Specifications:

    Unit ramp steady-state error: 5%

    Crossover frequency: $\omega_{gc} = 2$ rad/s

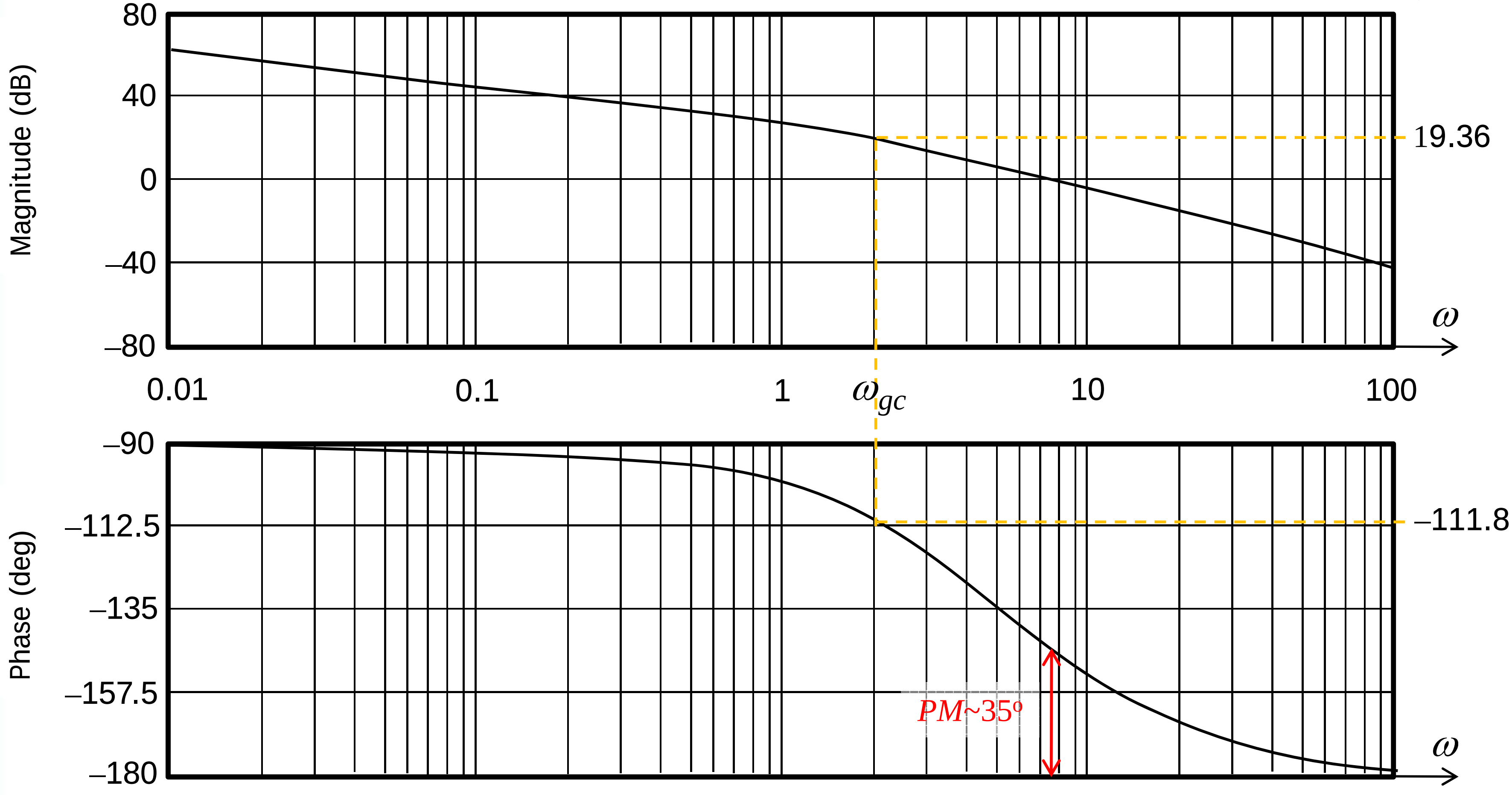
    Phase margin: $PM = 40^{\circ}$
  - Open-loop poles: $s = 0, -5$.

Lag Compensator

- Example cont'd
  - Gain $a_0 = 10$ meets steady-state error spec.
  - Next, draw Bode plots for $a_0 G_p(j\omega)$.
  - At $\omega = \omega_{gc} = 2$ rad/s we find $a_0 M_G = 9.28$ and $\theta_G = -111.8^\circ$.
  - The above design equations yield $a_1 = 8.185$ and $b_1 = 8.892$.
  - Controller: $G_c(s) = 10 \dfrac{0.82s + 1}{8.89s + 1}$,

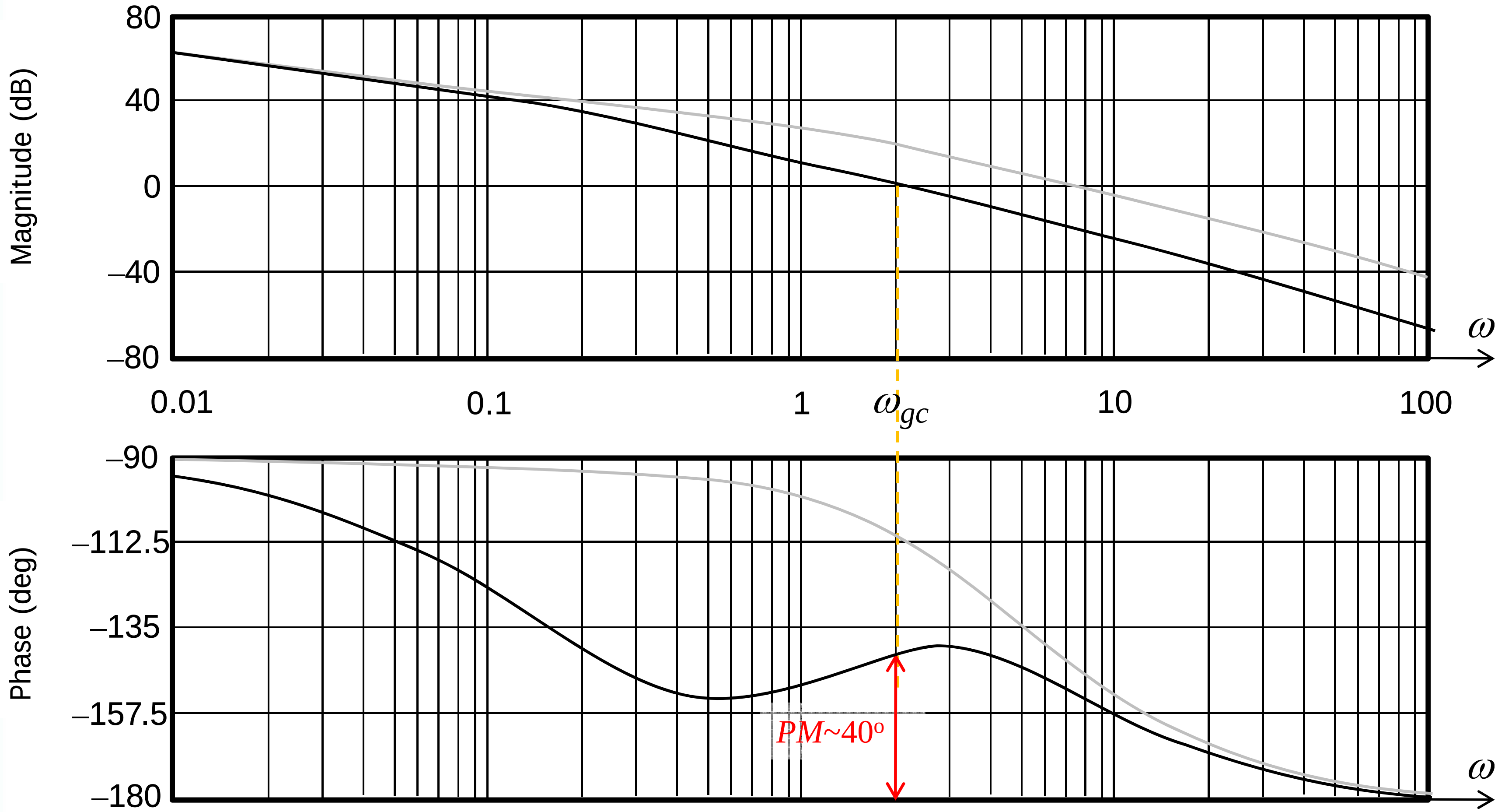
Lag Compensator

Bode plots for uncompensated open-loop system with a_0



Lag Compensator

Bode plots for compensated open-loop system



1^{st}-Order Controllers

- Lead or Lag Compensation?
  - To stabilise the closed-loop system given a stable open-loop system use either lead or lag comp.
  - To stabilise the closed-loop system when given an unstable open-loop system use a lead comp.
  - If the desired modulus crossover frequency ω_{gc} is larger than that of the plant use lead compensator. If smaller use a lag compensator.

Tutorial Exercises & Homework

- Tutorial Exercises
  - To be announced at the beginning of the tut session.
- Homework
  - None

Conclusion

- Phase-Lag Compensator Design Methods
- Phase-Lag Compensator Example
- Tutorial Exercises & Homework

Next Attraction! – Miss It & You'll Miss Out!

- Digital Control System Design (Burns, Chapter 7)

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Thank you!

Any Questions?