CONTROL I

ELEN3016

Zero-Error Systems

(Lecture 19)

Overview

- First Things First!
- Performance Indices
- Zero-Error Systems
- Optimum Zero-Error Systems
- Design Example
- Tutorial Exercises & Homework

- Next Attraction!

First Things First!

- Semester Test – Postmortem
  - Hints for the root locus in Question 2 contained in the Lecture 13's slide entitled "Variations on the RH/RL-Theme."
- Deadline for Presenting Lab Findings
  - Date agreed on: 9 October 2012

Zero-Error Systems

- n^{th} Order *Closed-Loop* Transfer Function

$$\frac{C(s)}{R(s)} = \frac{A_m s^m + A_{m-1} s^{m-1} + \ldots + A_2 s^2 + A_1 s + A_0}{B_n s^n + B_{n-1} s^{n-1} + \ldots + B_2 s^2 + B_1 s + B_0}$$

- n^{th} Order Zero-Error-Position System

$$\frac{C(s)}{R(s)} = \frac{B_0}{B_n s^n + B_{n-1} s^{n-1} + \ldots + B_2 s^2 + B_1 s + B_0}$$

Zero-Error Systems

- n^{th} Order *Closed-Loop* Transfer Function

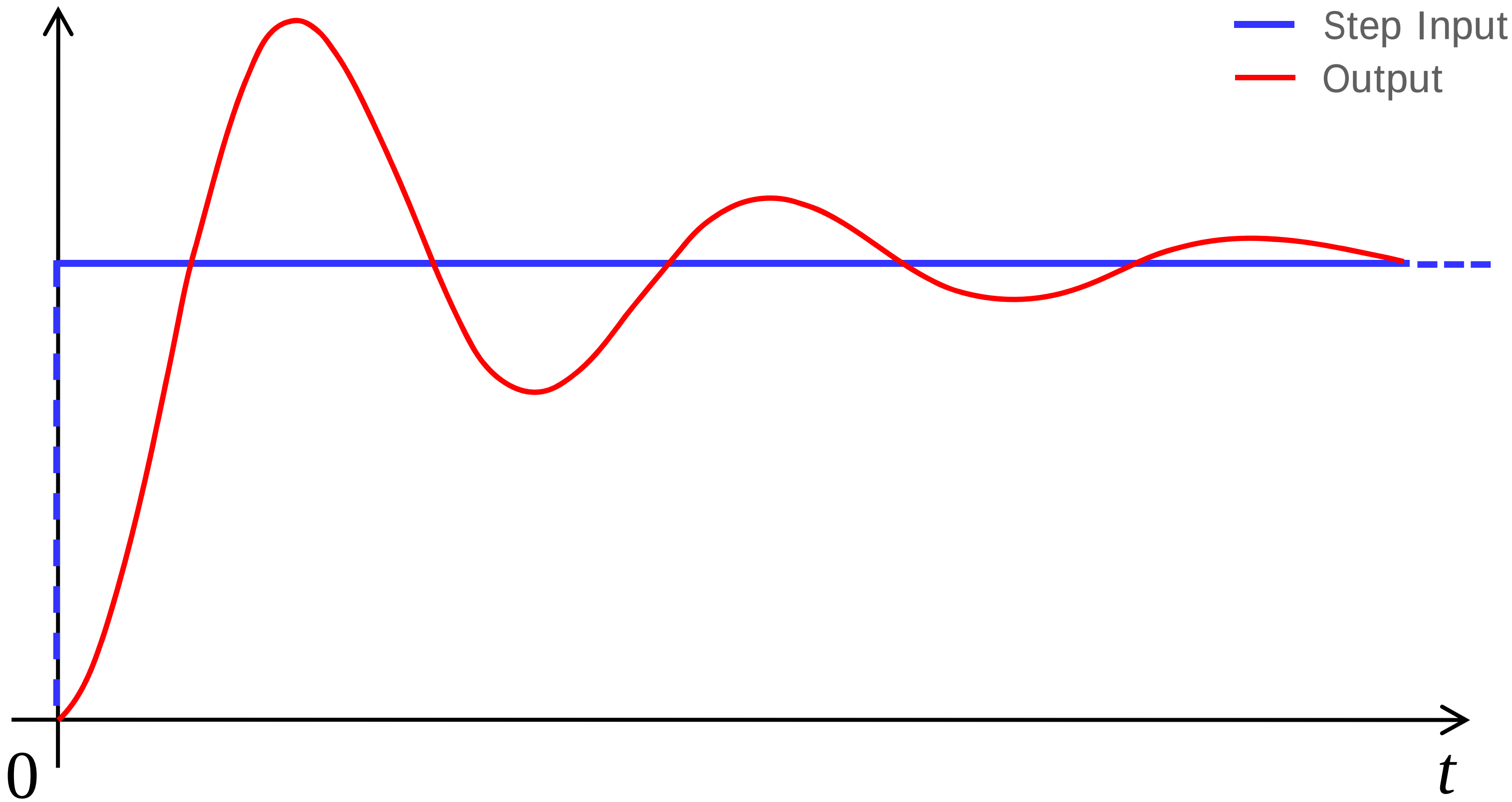
$$\frac{C(s)}{R(s)} = \frac{A_m s^m + A_{m-1} s^{m-1} + \ldots + A_2 s^2 + A_1 s + A_0}{B_n s^n + B_{n-1} s^{n-1} + \ldots + B_2 s^2 + B_1 s + B_0}$$

- n^{th} Order Zero-Error-Ramp System

$$\frac{C(s)}{R(s)} = \frac{B_1 s + B_0}{B_n s^n + B_{n-1} s^{n-1} + \ldots + B_2 s^2 + B_1 s + B_0}$$

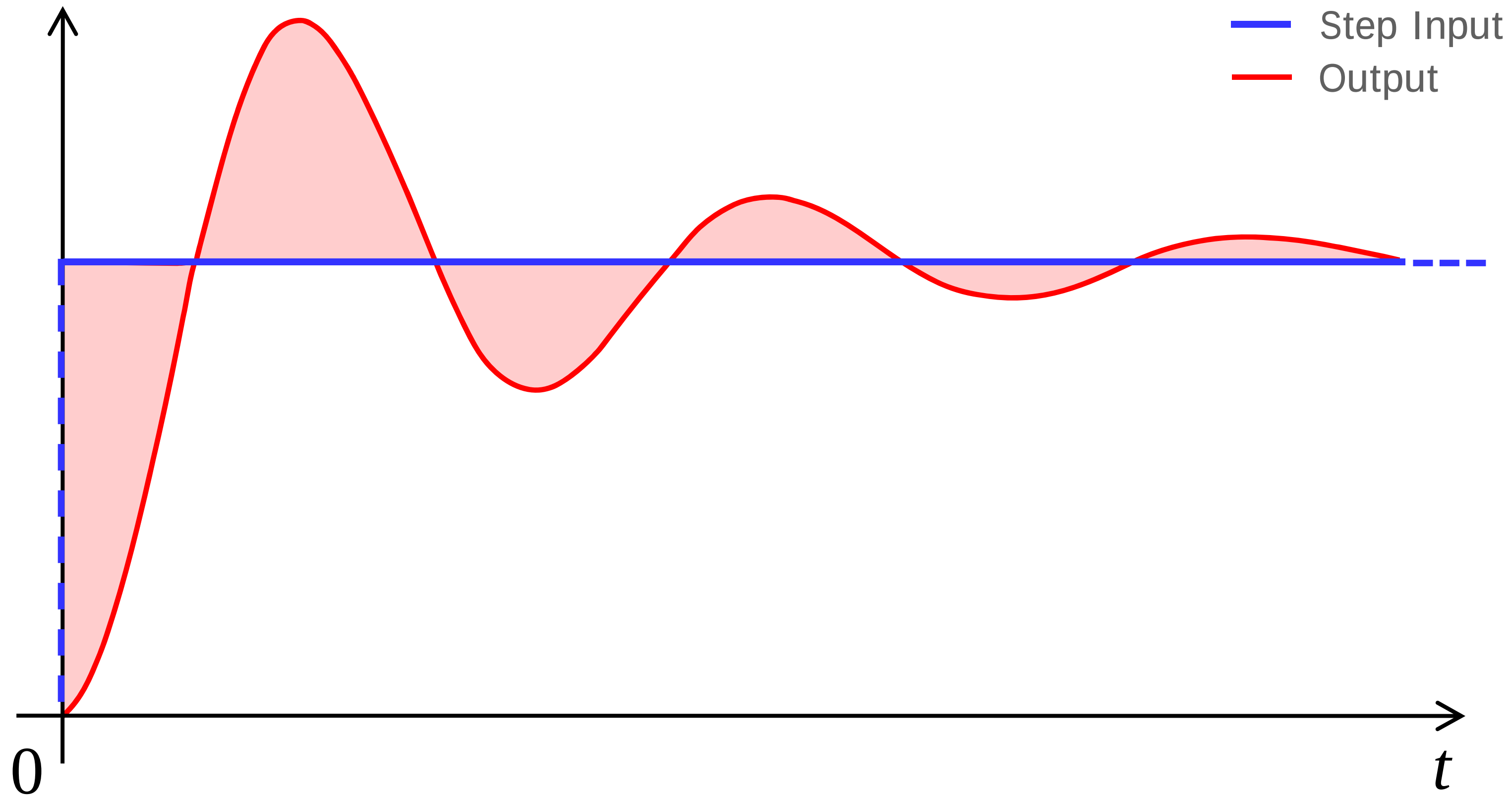
Performance Indices

- System Input and Output



Performance Indices

- Error-Area between Input and Output



Performance Indices

- Popular Performance Indices
  - Integrated Absolute Error (IAE):

$$S_1 = \int_0^\infty |e(t)|\,dt$$

  - Integrated Squared Error (ISE):

$$S_2 = \int_0^\infty e(t)^2\,dt$$

  - Integrated Time times Absolute Error (ITAE):

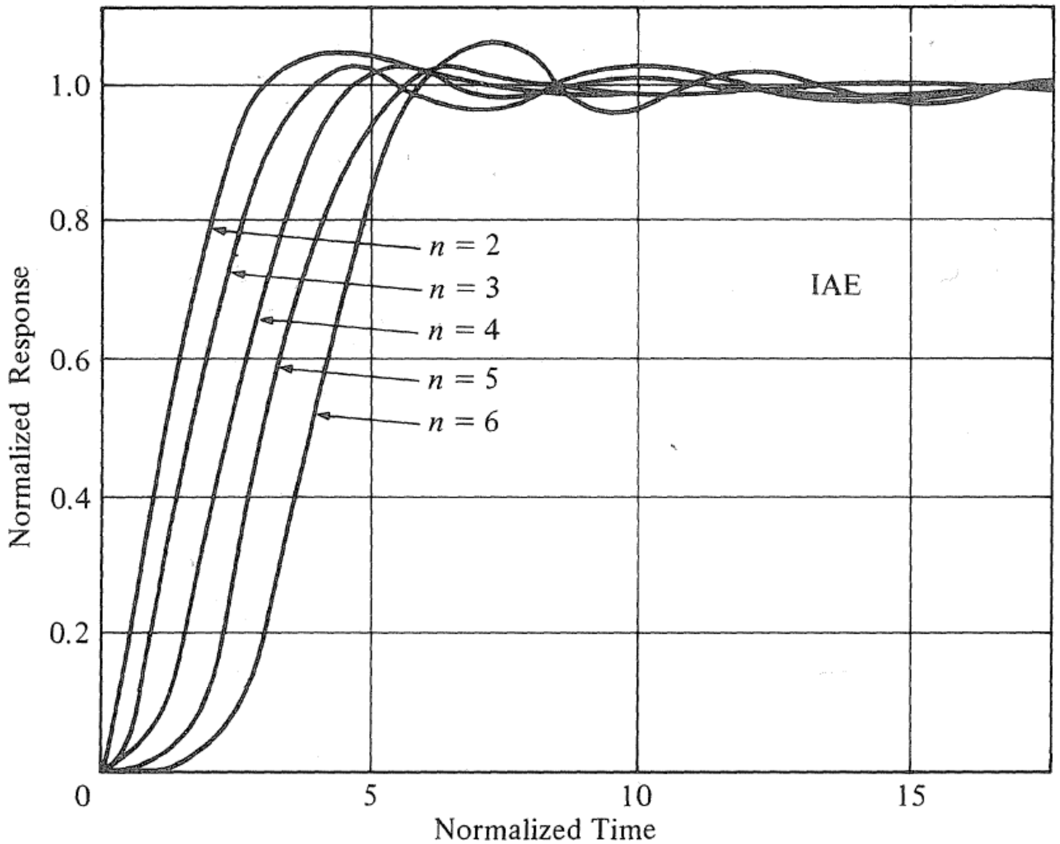
$$S_3 = \int_0^\infty t|e(t)|\,dt$$

Performance Indices

- Observation:
  - The smaller the deviation of the output from the input the smaller the error is as well as all of these performance indices.
  - It would be ideal if the error is zero but practically we strive to minimise the error.
  - For a given system we insert a controller, close the loop and select the controller to minimise the chosen performance index.

Performance Indices

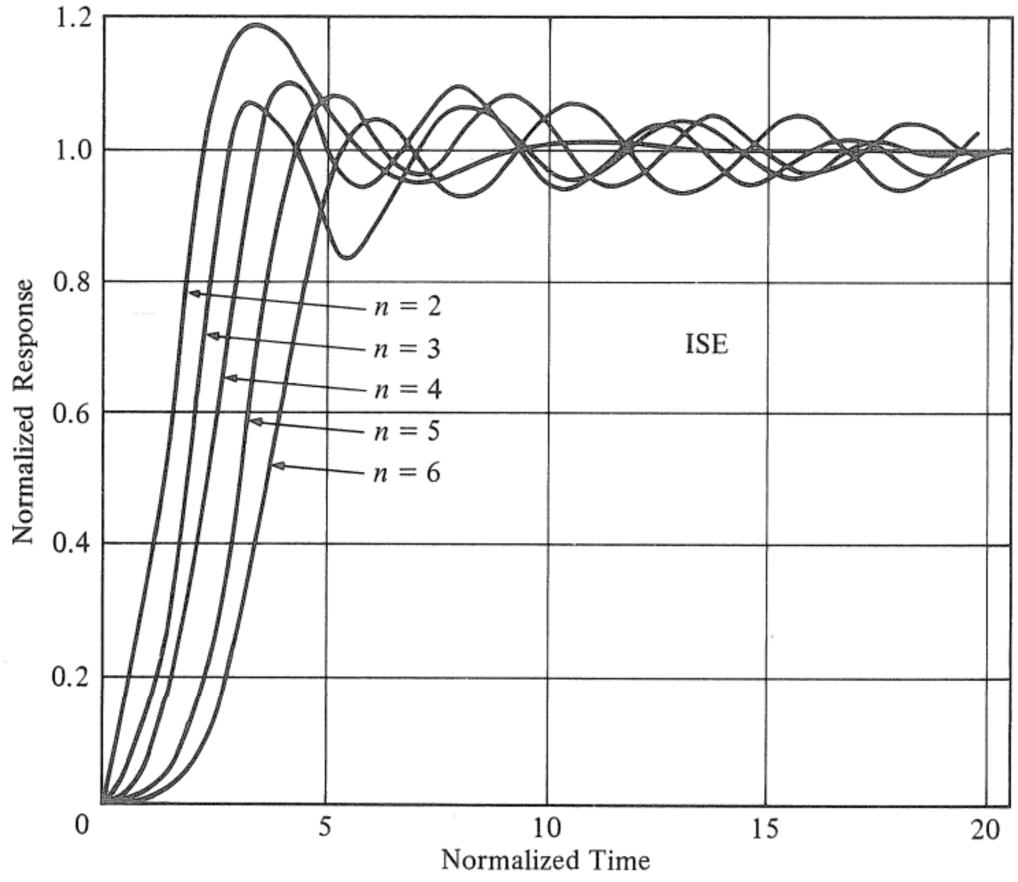
- n^{th} Order IAE-Optimum Step Responses



$$S_1 = \int_0^\infty |e(t)|\,dt$$

Performance Indices

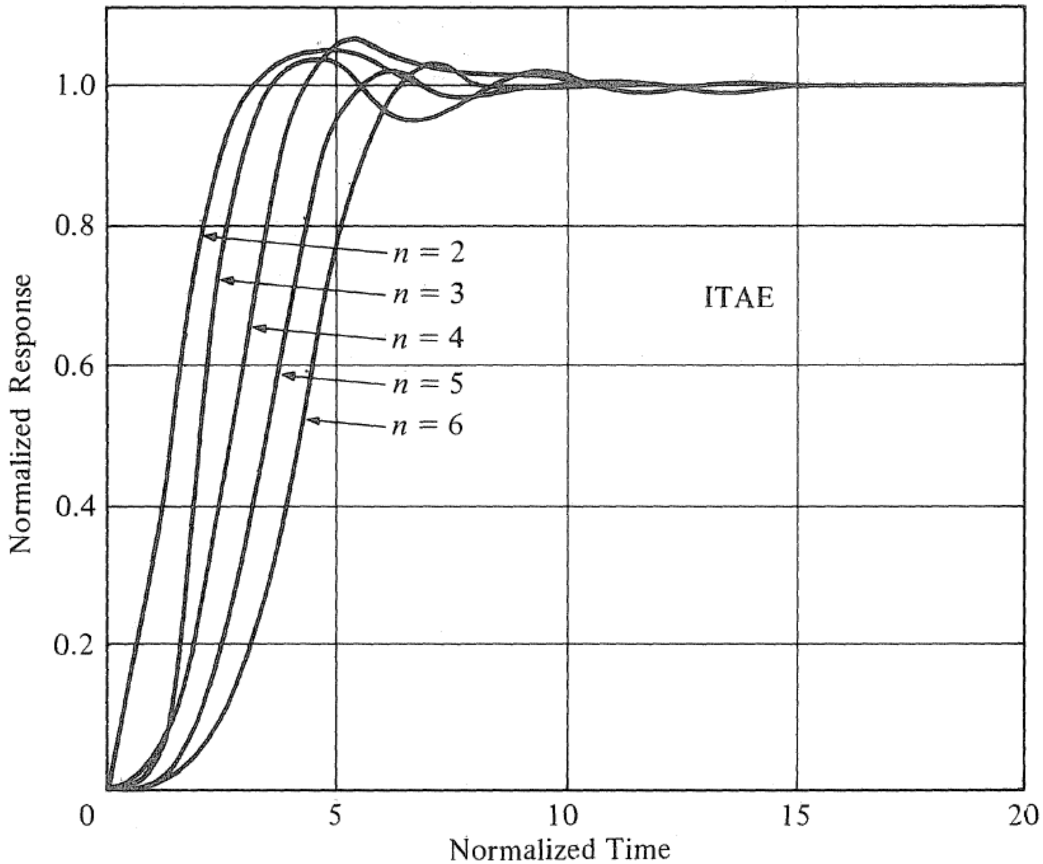
- n^{th} Order ISE-Optimum Step Responses



$$S_2 = \int_0^{\infty} e(t)^2\,dt$$

Performance Indices

- n^{th} Order ITAE-Optimum Step Responses



$$S_3 = \int_0^\infty t\left|e(t)\right|dt$$

Optimum Zero-Error Systems

- ITAE-Optimum ZEPS's Denominators:

$$n=1: \quad s+\omega_n$$

$$n=2: \quad s^2+1.414\omega_n s+\omega_n^2$$

$$n=3: \quad s^3+1.75\omega_n s^2+2.15\omega_n^2 s+\omega_n^3$$

$$n=4: \quad s^4+2.1\omega_n s^3+3.4\omega_n^2 s^2+2.7\omega_n^3 s+\omega_n^4$$

$$n=5: \quad s^5+2.8\omega_n s^4+5.0\omega_n^2 s^3+5.5\omega_n^3 s^2+3.4\omega_n^4 s+\omega_n^5$$

Optimum Zero-Error Systems

- ITAE-Optimum ZERS's Denominators:

$$n=2: \quad s^2+3.2\omega_n s+\omega_n^2$$

$$n=3: \quad s^3+1.75\omega_n s^2+3.25\omega_n^2 s+\omega_n^3$$

$$n=4: \quad s^4+2.41\omega_n s^3+4.93\omega_n^2 s^2+5.14\omega_n^3 s+\omega_n^4$$

$$n=5: \quad s^5+2.19\omega_n s^4+6.50\omega_n^2 s^3+6.30\omega_n^3 s^2+$$

$$+5.24\omega_n^4 s+\omega_n^5$$

Optimum Zero-Error Systems

- Example

The movie industry uses a dual-camera system which allows live actors to appear as if performing inside complex miniature sets.

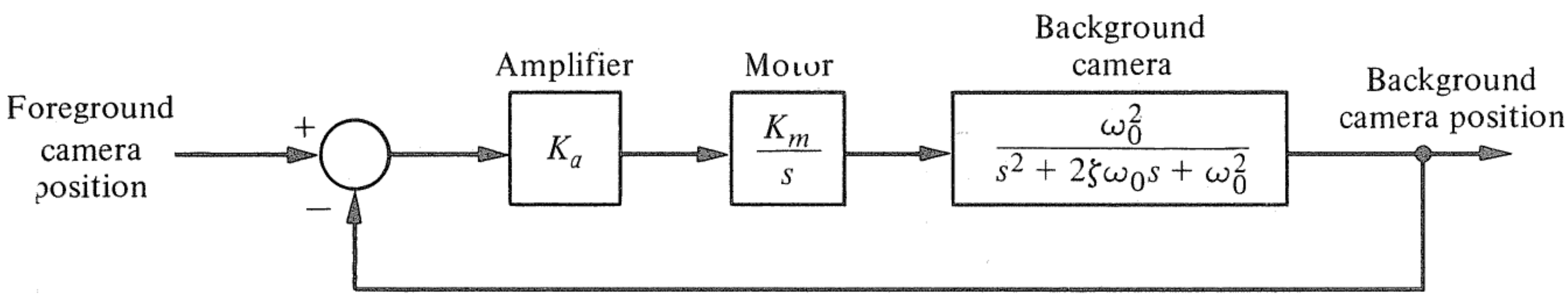
The *foreground* camera tracks the actors performing on a blue cyclorama stage while the *background* camera needs to track the foreground camera very accurately to reproduce all movements of the foreground camera in the scale of the miniature set.

During filming of a scene the foreground video is then super-imposed onto the background video.

Optimum Zero-Error Systems

- Example cont'd

  The simplified model of the dual-camera system including the master servo and slaving servo systems is shown below.



  – Design a controller to yield a ITAE-optimum Zero-Error-Position (closed-loop) system.

Optimum Zero-Error Systems

- Example cont'd
  - The closed-loop transfer function is

$$T(s) = \frac{K_a K_m \omega_0^2}{s^3 + 2\zeta\omega_0 s^2 + \omega_0^2 s + K_a K_m \omega_0^2}.$$

  - Comparing the 3rd order denominator in the table for ITAE-optimum Zero-Error-Position systems with the closed-loop system's denominator we obtain

$$2\zeta\omega_0 = 1.75\omega_n, \quad \omega_0^2 = 2.15\omega_n^2, \quad K_a K_m \omega_0^2 = \omega_n^3.$$

Optimum Zero-Error Systems

- ITAE-Optimum ZEPS's Denominators:

$$n=1: \quad s+\omega_n$$

$$n=2: \quad s^2+1.414\omega_n s+\omega_n^2$$

$$n=3: \quad s^3+1.75\omega_n s^2+2.15\omega_n^2 s+\omega_n^3$$

$$n=4: \quad s^4+2.1\omega_n s^3+3.4\omega_n^2 s^2+2.7\omega_n^3 s+\omega_n^4$$

$$n=5: \quad s^5+2.8\omega_n s^4+5.0\omega_n^2 s^3+5.5\omega_n^3 s^2+3.4\omega_n^4 s+\omega_n^5$$

Tutorial Exercises & Homework

- Example cont'd
  - For rapid response a large value for ω_n is selected and a settling time of less than one second. We choose $\omega_n = 50$ rad/s and $\zeta = 0.60$.

$$K_a K_m = \frac{\omega_n^3}{\omega_0^2} = \frac{\omega_n^3}{2.15\omega_n^2} = \frac{\omega_n}{2.15} = 23.2$$

  - The closed-loop transfer function & settling time

$$T(s) = \frac{125\,000}{s^3 + 87.5s^2 + 5\,375s + 125\,000}, \quad t_s = 0.15\,\text{s}\,.$$

Tutorial Exercises & Homework

- Tutorial Exercises
  - Prove that the steady-state error for a step-excited zero-error position system is zero. (Hint: Express the open-loop transfer function i.t.o. the closed-loop transfer function and substitute the expression for a zero-error position system in.)
  - Prove that the steady-state error for a ramp-excited zero-error ramp system is zero.
- Homework
  - None

Conclusion

- Performance Indices
- Zero-Error Systems
- Optimum Zero-Error Systems
- Design Example
- Tutorial Exercises & Homework

Next Attraction! – Miss It & You'll Miss Out!

- Classical Design in the Frequency Domain Continued (Burns, Chapter 6)

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Thank you!

Any Questions?