CONTROL I

ELEN3016

Classical Design in the Frequency Domain

(Lecture 16)

Overview

- First Things First!
- Nyquist Stability Criterion
- Tutorial Exercises & Homework

- Next Attraction!

First Things First!

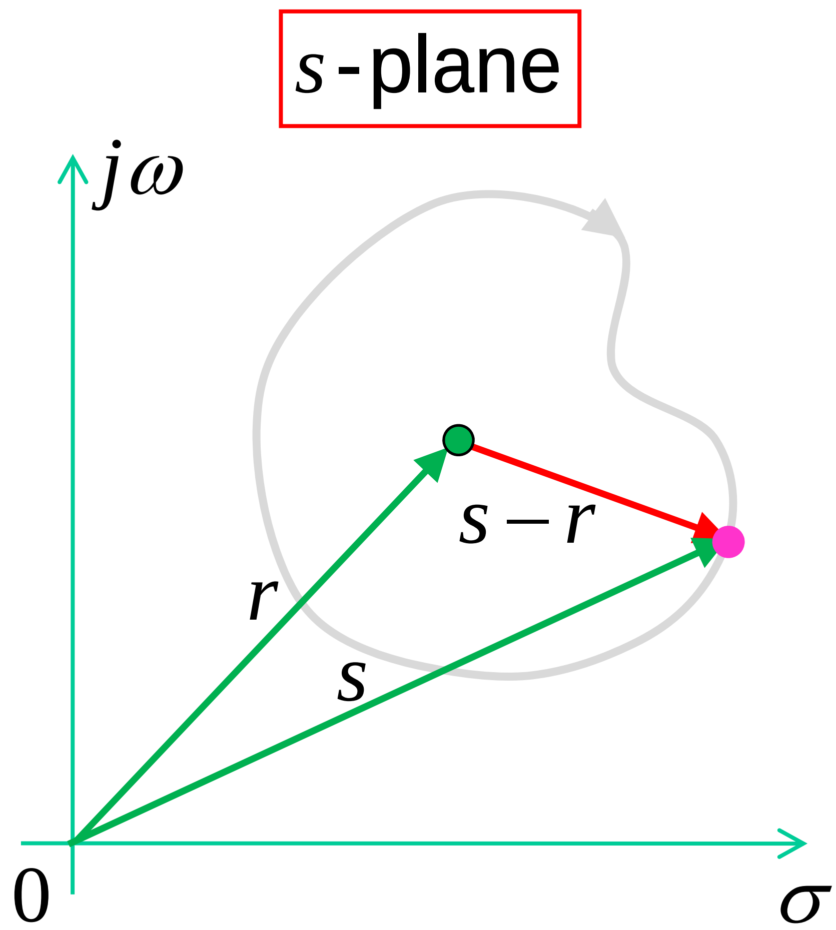
- Deadline for presenting your lab findings
  - We agreed on: 9 October 2012
- Tutorial session
  - Thursday 23 August 2012

Nyquist Stability Criterion

- Objective: Determine closed-loop stability.
- For the function $F(s) = s - r$, in the s-plane consider two closed contours; one enclosing r and one not enclosing r.
- Study the *net change in angle* of the vector $s - r$ as we traverse these contours starting at an arbitrary point going around once and back to this point.

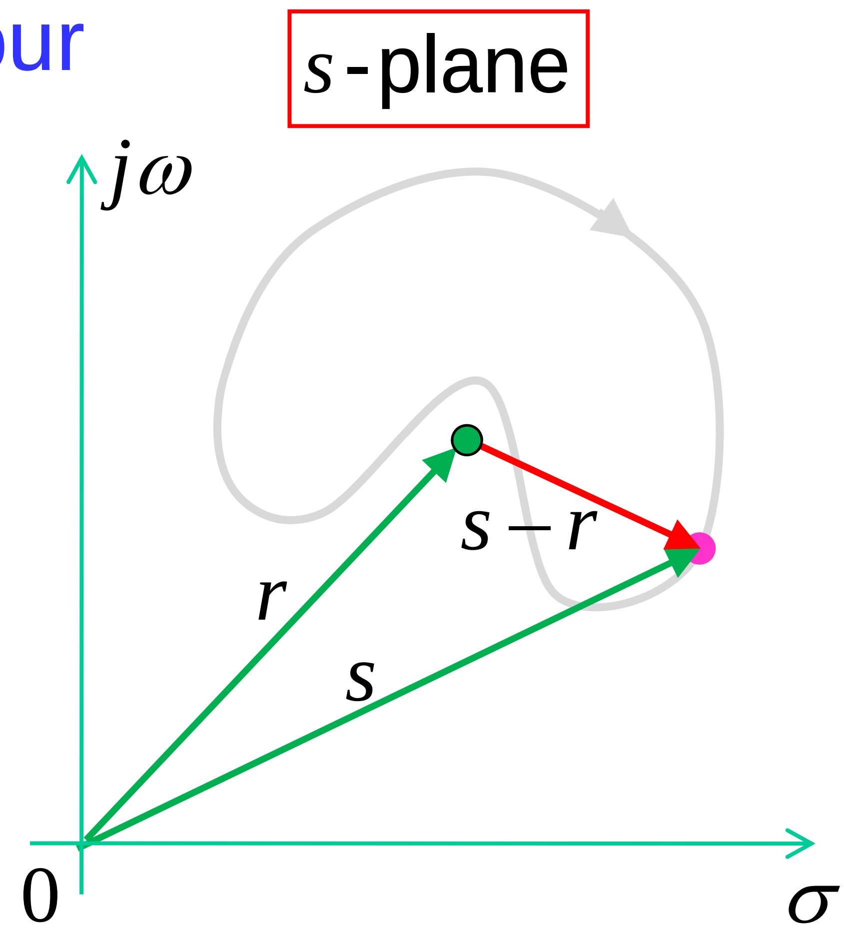
Nyquist Stability Criterion

- Enclosing Contour
  - Each complete clockwise lap travelled increases the angle $\angle(s-r)$ by 2π.



Nyquist Stability Criterion

- Non-enclosing Contour
  - Now a complete lap does not change $\angle(s-r)$.



Nyquist Stability Criterion

- For the function $F(s)=\dfrac{1}{s-r}$, in the s-plane consider two closed contours; one enclosing r and one not enclosing r.
- Concluding about the net change in $\angle\dfrac{1}{s-r}$ is straightforward since $\angle\dfrac{1}{s-r}=-\angle(s-r)$.
- **Conclusion:** Here a lap along an enclosing contour <u>decreases</u> $\angle\dfrac{1}{s-r}$ by 2π while a non-enclosing contour yields no net change.

Nyquist Stability Criterion

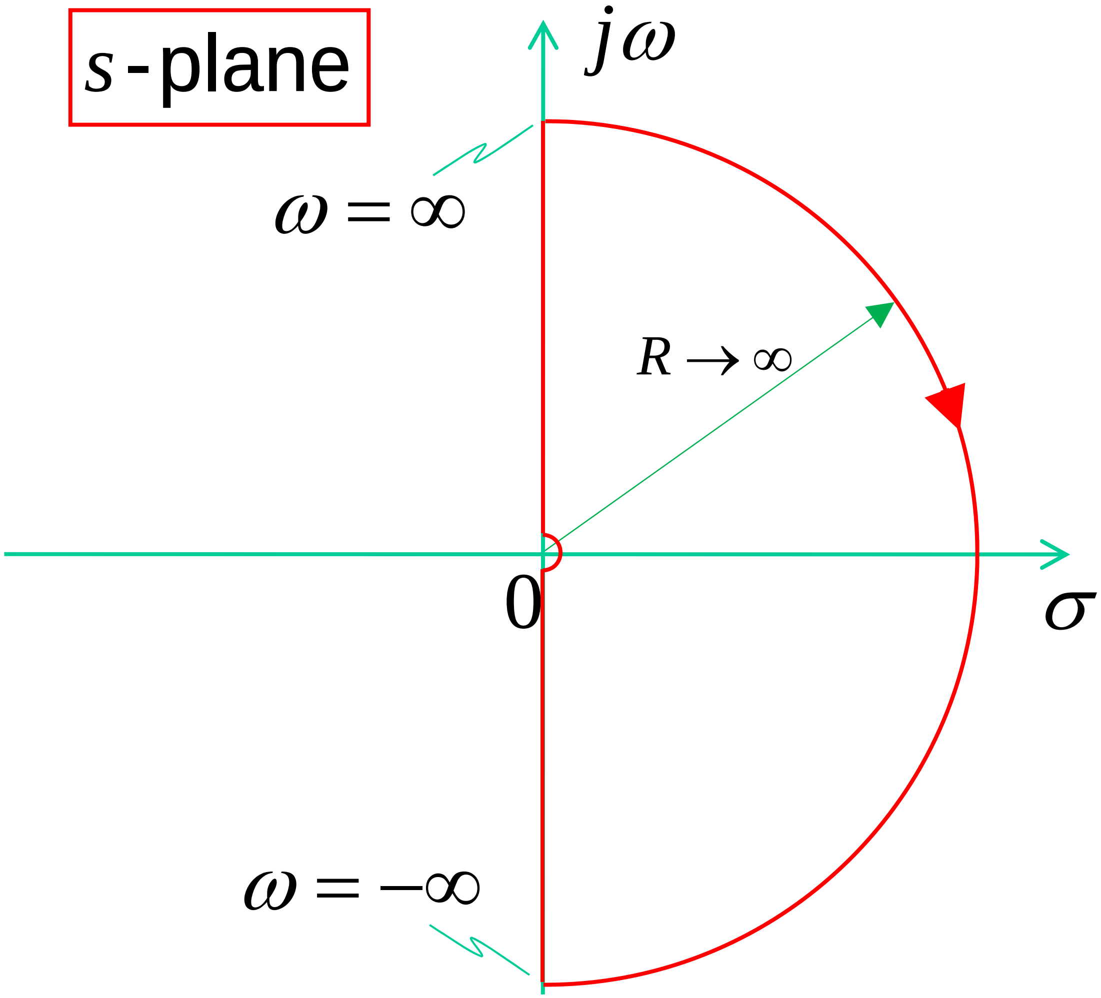
– Now consider the function

$$F(s) = \frac{(s - z_1)(s - z_2)...(s - z_m)}{(s - p_1)(s - p_2)...(s - p_n)}$$

If a contour encloses Z zeros and P poles of $F(s)$, the net no. of encirclements of the origin of the $F(s)$-plane is $N = Z - P$.

– Choose the contour to be the semi-circle that encloses the RHP but excludes the LHP.

Nyquist Stability Criterion



Nyquist Stability Criterion

– Closed-loop characteristic equation:

$$F(s) = 1 + GH(s) = 1 + \frac{N_G N_H(s)}{D_G D_H(s)} = 0$$

$$\frac{D_G D_H(s) + N_G N_H(s)}{D_G D_H(s)} = 0$$

– Zeros of $F(s)$ are all the closed-poles.

– Poles of $F(s)$ are all the open-loop poles.

Nyquist Stability Criterion

- Nyquist Criterion
  - With the contour enclosing the RHP only:

$$Z = N + P$$

- Z: No. of closed-loop poles in the RHP.
- N: Net no. of encirclements of the origin.
- P: No. of open-loop poles in the RHP.

Question: What happens if we redefine $F(s) = G(s)H(s)$?

Nyquist Stability Criterion

- Nyquist Criterion for $F(s)=G(s)H(s)$
  - With the contour enclosing the RHP <u>only</u>:

$$Z = N + P$$

- P: No. of open-loop poles in the RHP.
- N: Net no. of encirclements of the point -1.
- Z: No. of closed-loop poles in the RHP.

Tutorial Exercises & Homework

- Tutorial Exercises
  - None.
- Homework
  - Study all relevant sections in Burns.

Conclusion

- Nyquist Stability Criterion
- Tutorial Exercises & Homework

Next Attraction! – Miss It & You'll Miss Out!

- Classical Design in the Frequency Domain Continued (Burns, Chapter 6)

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Thank you!

Any Questions?