

CONTROL I

ELEN3016

Classical Design in the Frequency Domain

(Lecture 15)

Overview

- First Things First!
- Frequency Response Representations
- Tutorial Exercises & Homework
- **Next Attraction!**

First Things First!

- None

Frequency Response Representations

- Polar Plot

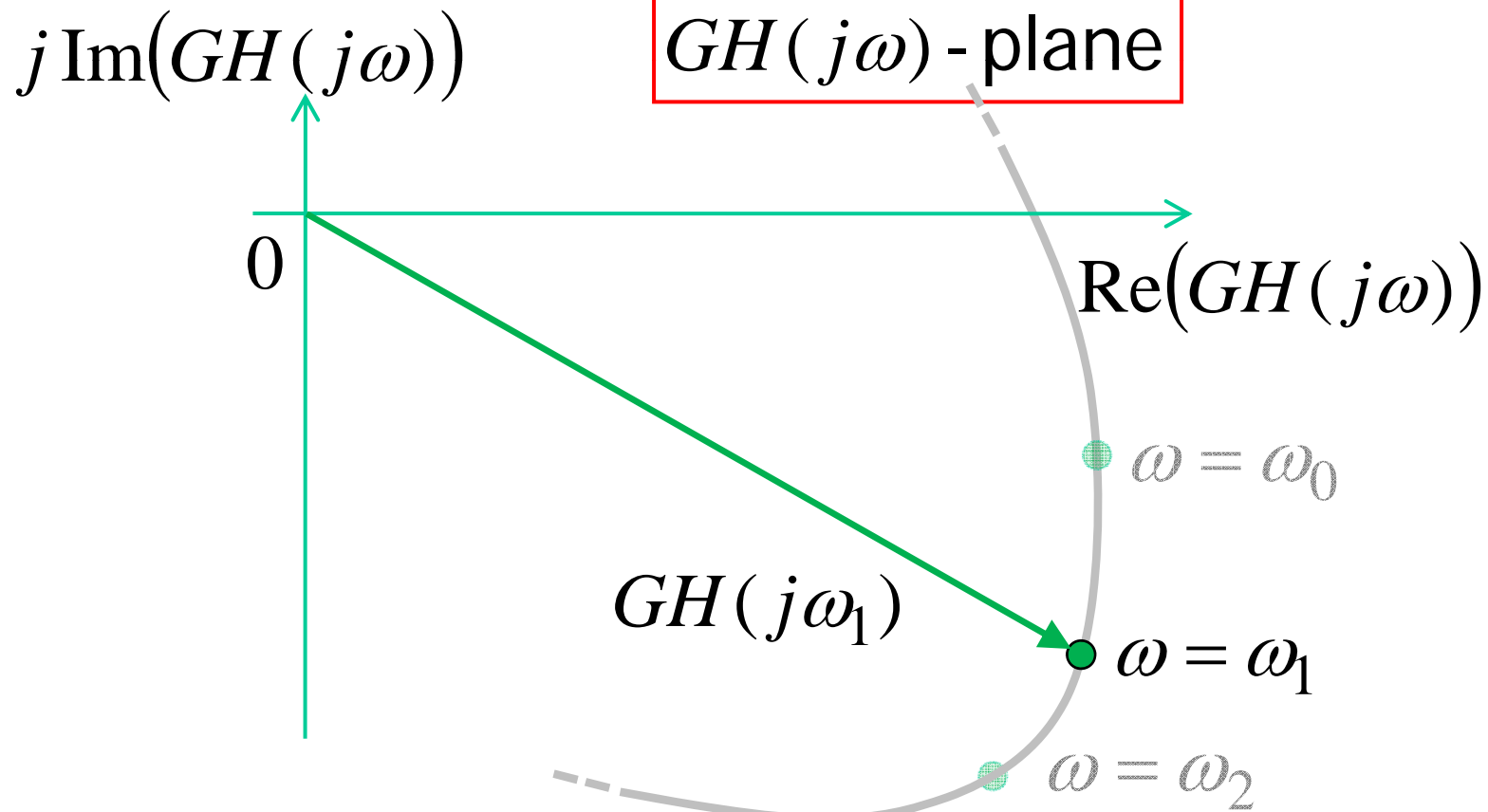
- ω (frequency) is the independent variable.
- Plot $G(j\omega)H(j\omega) = G(s)H(s)|_{s=j\omega}$ as a function of ω in the complex plane called the $G(j\omega)H(j\omega)$ -plane since

$$GH(j\omega) = \text{Re}(GH(j\omega)) + j \text{Im}(GH(j\omega))$$

where $GH(j\omega) \equiv G(j\omega)H(j\omega)$.

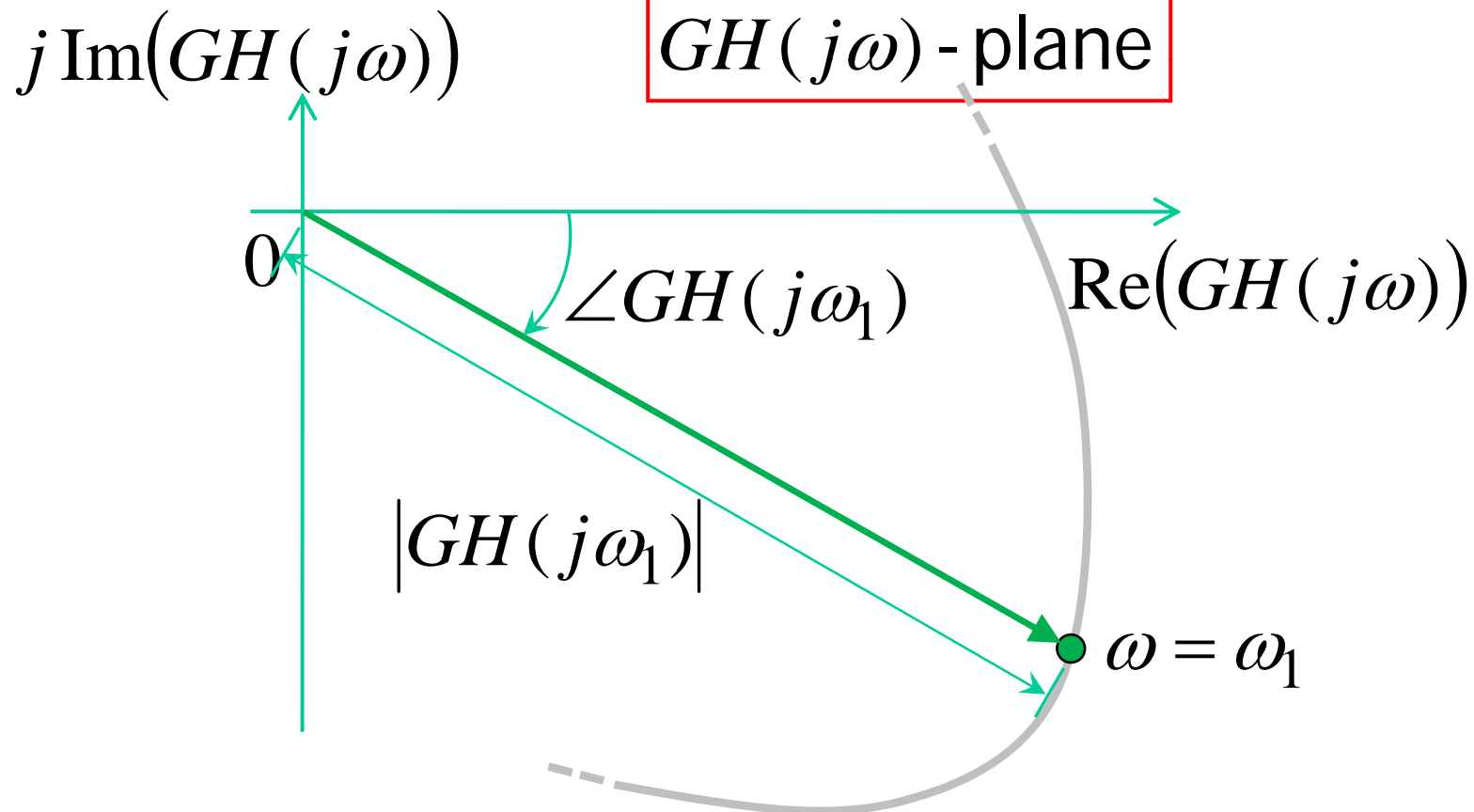
Frequency Response Representations

- Polar Plot – Illustration



Frequency Response Representations

- Polar Plot – Illustration



Frequency Response Representations

- Example 1

- First-order system, $G(s)H(s) = \frac{1}{1 + \tau s}$, $\tau > 0$
- Frequency response:

$$\begin{aligned} GH(j\omega) &= \frac{1}{1 + j\tau\omega} \times \frac{1 - j\tau\omega}{1 - j\tau\omega} \\ &= \frac{1 - j\tau\omega}{1 + \tau^2\omega^2} \end{aligned}$$

Frequency Response Representations

- Example 1 cont'd

- Magnitude:

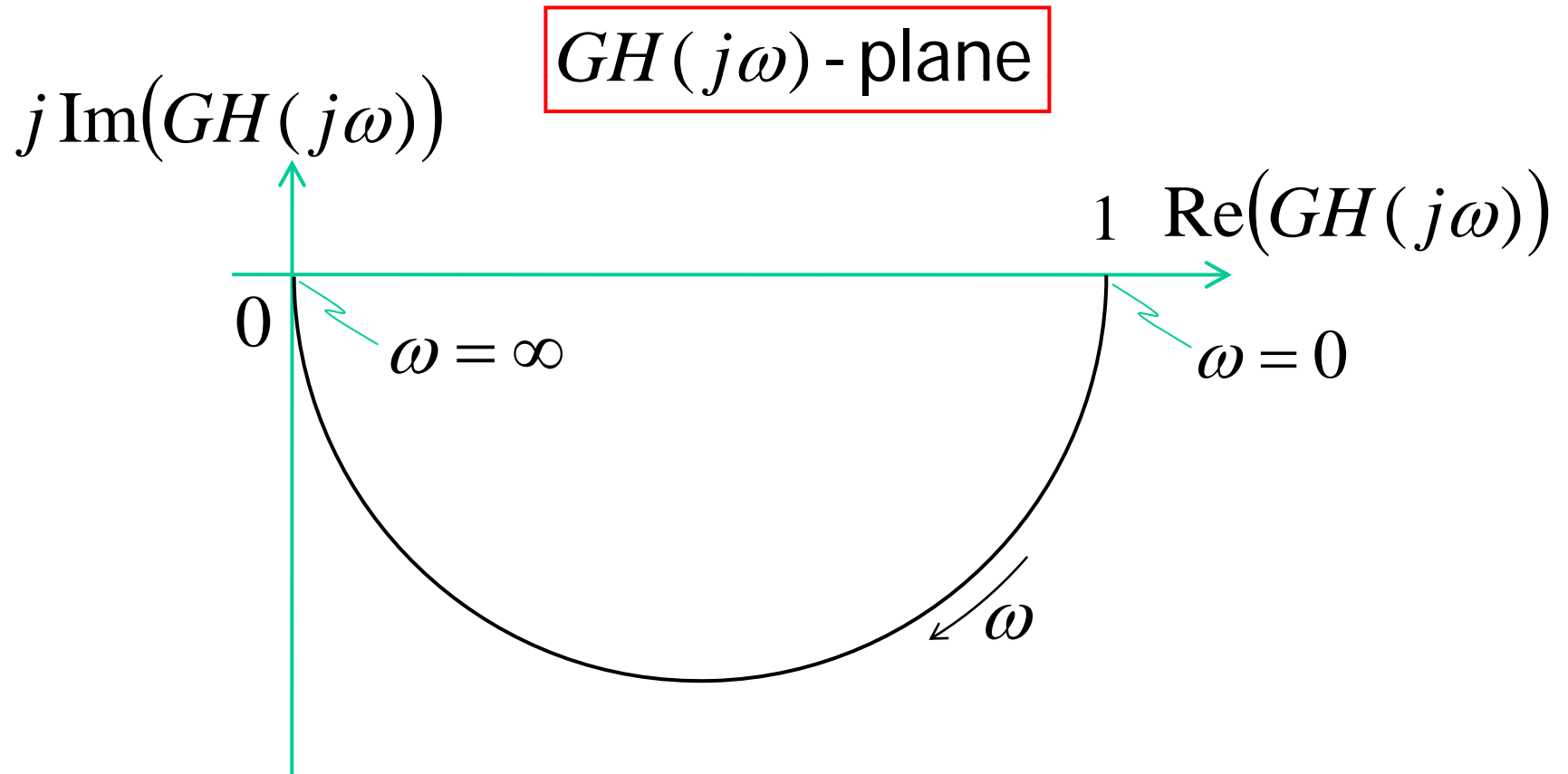
$$|GH(j\omega)| = \sqrt{\left(\frac{1}{1+\tau^2\omega^2}\right)^2 + \left(\frac{\tau\omega}{1+\tau^2\omega^2}\right)^2} = \frac{1}{1+\tau^2\omega^2}$$

- Phase:

$$\angle GH(j\omega) = \arctan \frac{-\tau\omega}{1}$$

Frequency Response Representations

- Example 1 cont'd



Frequency Response Representations

- Example 2

- Second-order system, $GH(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
- Frequency response:

$$\begin{aligned} GH(j\omega) &= \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2} \\ &= \frac{\omega_n^2}{(\omega_n^2 - \omega^2) + j2\zeta\omega_n\omega} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\left(\frac{\omega}{\omega_n}\right)} \end{aligned}$$

Frequency Response Representations

- Example 2 cont'd

- Magnitude:

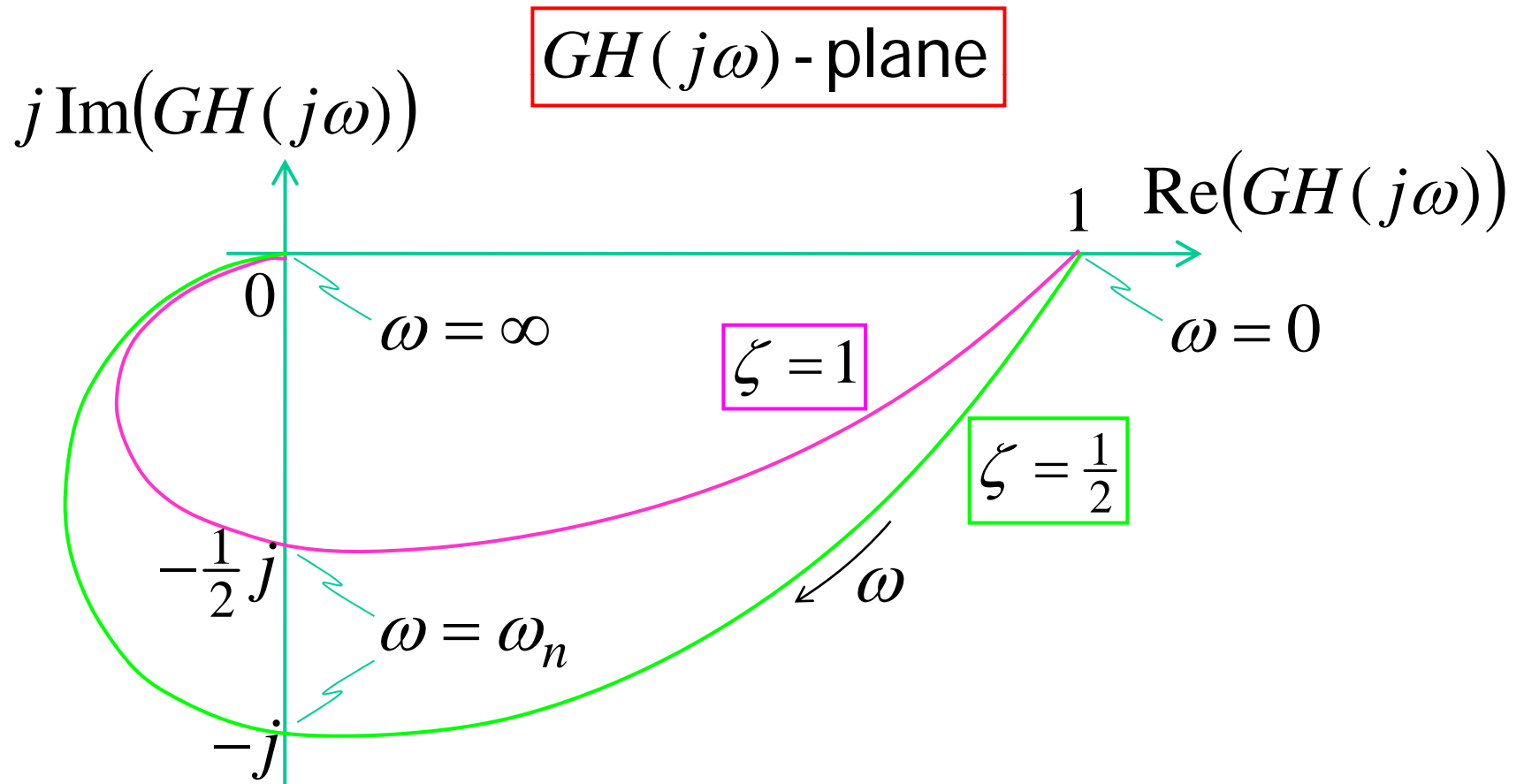
$$|GH(j\omega)| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\zeta^2 \left(\frac{\omega}{\omega_n}\right)^2}}$$

- Phase:

$$\angle GH(j\omega) = \arctan \frac{-2\zeta \left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

Frequency Response Representations

- Example 2 cont'd



Frequency Response Representations

- Example 3

- 2nd order, type 1 system, $GH(s) = \frac{1}{s(1 + \tau s)}$

- Frequency response:

$$\begin{aligned} GH(j\omega) &= \frac{1}{j\omega(1 + j\tau\omega)} = \frac{1}{-\tau\omega^2 + j\omega} = \dots \\ &= \frac{-(\tau\omega^2 + j\omega)}{\tau^2\omega^4 + \omega^2} = \frac{-(\tau\omega + j)}{\tau^2\omega^3 + \omega} \end{aligned}$$

Frequency Response Representations

- Example 3 cont'd

- Magnitude & Phase:

$$\begin{aligned}\omega = 0^+: \quad GH(j\omega)\Big|_{\omega=0^+} &= \frac{1}{j\omega}\Big|_{\omega=0^+} \times \frac{1}{1+j\tau\omega}\Big|_{\omega=0^+} \\ &= (+\infty)e^{-j\frac{\pi}{2}} \times e^{-j0^+} \\ &= (+\infty)e^{-j\frac{\pi}{2}}\end{aligned}$$

Frequency Response Representations

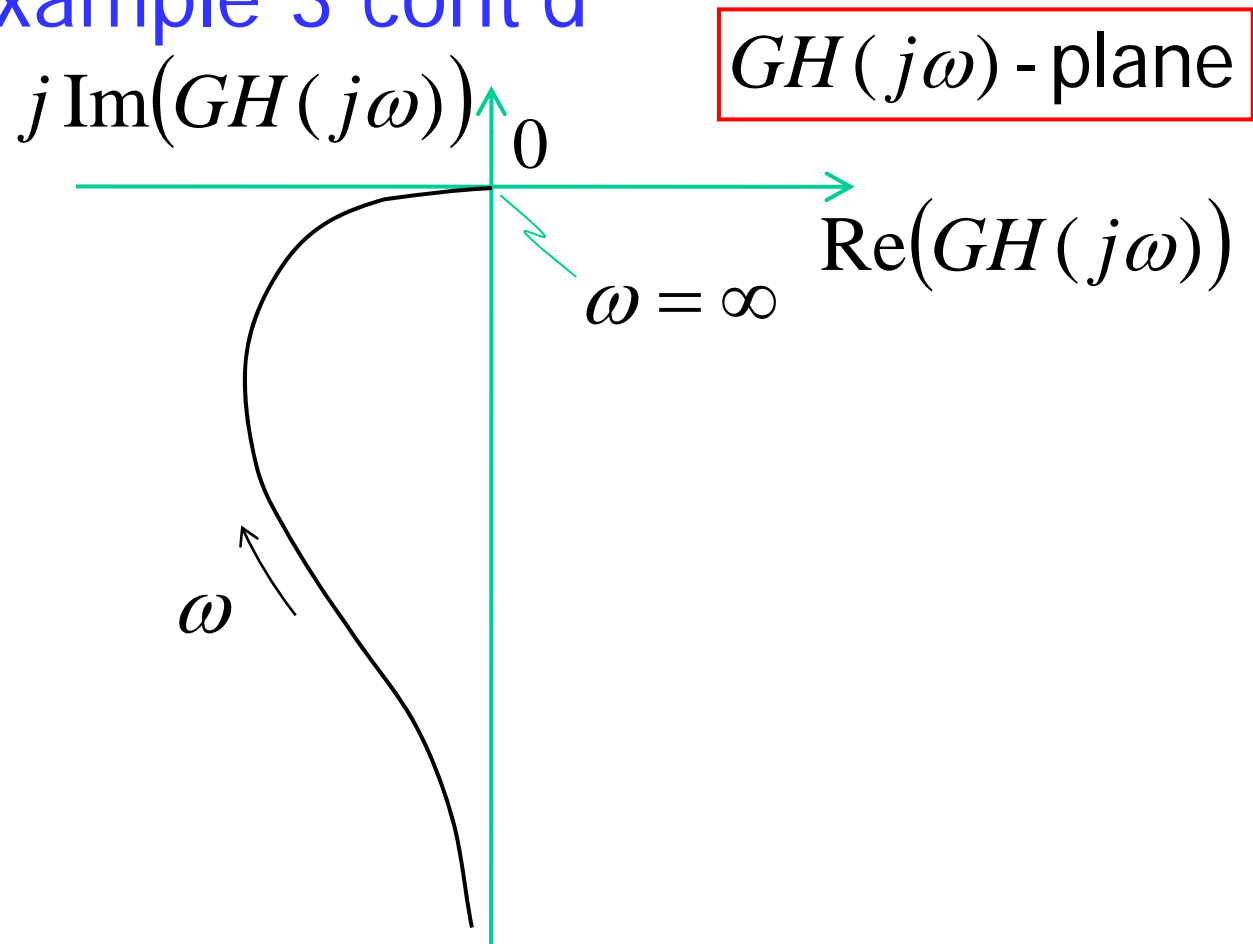
- Example 3 cont'd

- Magnitude & Phase:

$$\begin{aligned}\omega = +\infty: \quad GH(j\omega)\Big|_{\omega=+\infty} &= \frac{1}{j\omega}\Big|_{\omega=+\infty} \times \frac{1}{1+j\tau\omega}\Big|_{\omega=+\infty} \\ &= (0^+)e^{-j\frac{\pi}{2}} \times (0^+)e^{-j\frac{\pi}{2}} \\ &= (0^+)e^{-j\pi}\end{aligned}$$

Frequency Response Representations

- Example 3 cont'd



Tutorial Exercises & Homework

- Tutorial Exercises

- Draw the frequency polar plot for $G(s)H(s) = \frac{1}{s}$.

- Homework

- Study all relevant sections in Burns.


Conclusion

- Frequency Response Representations (Polar Plot)
- Three examples of polar plots
- Tutorial Exercises & Homework

Next Attraction! – Miss It & You'll Miss Out!

- Classical Design in the Frequency Domain Continued (Burns, Chapter 6)

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Thank you!
Any Questions?